Research Contribution 40

Equations for Predicting Height-to-Crown-Base, 5-Year Diameter-Growth Rate, 5-Year Height-Growth Rate, 5-Year Mortality Rate, and Maximum Size-Density Trajectory for Douglas-fir and Western Hemlock in the Coastal Region of the Pacific Northwest

by

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June 2003

Oregon State University
College of Forestry

Forest Research Laboratory

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Acknowledgments

The funds for developing the equations reported herein were provided by the Stand Management Cooperative, University of Washington, Seattle, Washington. The funds used to write and publish these results were provided by the College of Forestry, Oregon State University, Corvallis.

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Abstract

Hann, DW, DD Marshall, and ML Hanus. 2003. *Equations for Predicting Height-to-Crown-Base, 5-year Diameter-Growth Rate, 5-year Height-Growth Rate, 5-year Mortality Rate, and Maximum Size-Density Trajectory for Douglas-fir and Western Hemlock in the Coastal Region of the Pacific Northwest.* Research Contribution 40, Forest Research Laboratory, Oregon State University, Corvallis.

Using existing permanent research plot data, we developed equations for predicting height-to-crown-base (*HCB*), 5-yr diameter-growth rate (ΔD), 5-yr height-growth rate (ΔH), 5-yr mortality rate (*PM*), and the maximum size-density trajectory for Douglas-fir and western hemlock in the coastal region of the Pacific Northwest. With the exception of the *HCB* equation, the equations developed for predicting trees from untreated plots agreed in predictive behavior with previously published equations for the study area. The *HCB* equation predicted shorter *HCB* (and therefore longer crown lengths [*CL*]) than previously published equations for the study area.

Western hemlock showed no response to fertilization. Modifiers for fertilization response were incorporated into the final equations for predicting ΔD, ΔH, and *PM* in Douglas-fir. All three modifiers for Douglas-fir predicted an increase in growth and mortality rates with the amount of nitrogen applied and a decrease with number of years since fertilization, with most of the fertilization effect gone within 15 yr of application. For the ΔD and ΔH modifiers, the size of the increase varied by the site index (*SI*) of the plot, with plots of lower site quality showing greater increases. For ΔD, fertilization response did not appear to vary by plot density, tree size, or tree position within the plot.

Modifiers for thinning response were incorporated into the final equations for predicting tree ΔD for both species and ΔH for Douglas-fir. For both species, the ΔD thinning-effects modifier predicted an increased growth rate with the proportion of the *BA* removed and a decrease with years since thinning; most of the thinning effect was gone within 10 yr. For Douglas-fir, the ΔH

thinning-effects modifier predicted a reduced growth rate immediately after thinning, with the size of the reduction increasing with the intensity of thinning. Most of the reduction was gone by about 10 yr.

For Douglas-fir, the combined effect on ΔD and ΔH of applying both thinning and fertilization could be adequately characterized by the product of the thinning modifier and the fertilization modifier. The percent increase in predicted growth rate due to a combined treatment thus was greater than the sum of the percent increases for each treatment alone.

Analysis of the maximum size-density trajectory data strongly suggests that plots of neither species approach a single maximum stand density index value (*SDI*) as they develop. The potential yield for a given site therefore depends, not only on its *SI*, but also on its maximum *SDI*. Fertilization does not appear to affect the intercept of the maximum size-density line for Douglas-fir.

The strengths and weaknesses of the existing data sets and the modeling and analytical approaches tested during development of these equations are presented to aid future modelers, and alternative modeling approaches are explored.

Keywords: ORGANON Growth-and-Yield model, stand development, Stand Management Cooperative

A user's manual (Hann et al. 1997) has been prepared covering the SMC and other versions of ORGANON. It and the ORGANON software are available from the ORGANON web site: www.cof.orst.edu/cof/fr/research/organon.

Unit conversions

1 acre (ac)	=	4047 m^2
1 foot (ft)	=	0.305 meters (m)
1 inch (in.)	=	2.34 centimeters (cm)
1 pound (lb)	=	453.6 grams (gm)

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Introduction

Because of the unique combination of weather conditions and geographic features in the Pacific Northwest, its coniferous forests are among the most productive and ecologically complex in the world (Franklin and Dyrness 1973). This situation provides forest managers with both an opportunity to utilize the productivity of these unique forests for the maximum benefit of humanity and a challenge to do so while protecting the often delicate and complex ecosystems.

As a result of the high productivity of these forests, the manufacture of forest products has developed into a major industry in the region. Therefore, wise management of the forests of the Pacific Northwest is crucial. Accurate prediction of the development of forest stands is critical to all forest management activities (Hann and Bare 1979; Hann and Brodie 1980). For example, stand-development information is required for evaluating (1) the best silvicultural treatments for stands of trees to meet a host of alternative management objectives, (2) the potential allowable cut from forests, (3) the future availability of raw wood for manufacturing into wood products, and (4) quality of wildlife habitat and other values.

In the future, demands for accurate, precise growth-and-yield information in the Pacific Northwest will be much more exacting and comprehensive than in the past. Cutting of old-growth stands is fast coming to an end. As a result, future wood supplies will increasingly come either from intensively managed young-growth stands or from stands managed by the "new" forestry practices currently under development. In either case, the management of these stands will require good-quality growth-and-yield information on their response to alternative cutting strategies (such as thinning, shelterwood cutting, uneven-aged cutting, and patch cutting), fertilization, pruning, and genetic improvement. A particular concern is the effect of these treatments on wood quality (Barbour and Kellogg 1990).

Two of the major commercially valuable tree species in the Pacific Northwest are Douglas-fir [*Pseudotsuga menziesii* (Mirb.)

Franco] and western hemlock [*Tsuga heterophylla* (Raf.) Sarg.]. In 1990, information concerning stand development and resulting wood quality under alternative management activities for both of these species was either lacking or was based on data collected over a shorter duration than was available by 1990. For example, the most recently developed publicly available stand-development information for Pacific Northwest western hemlock was the natural stand yield tables of Wiley and Chambers (1981), and the widely used Douglas-fir growth-and-yield model, DFSIM (Curtis et al. 1981), was developed with data collected no later than 1974 (i.e., more than 15 yr of additional data had been collected since its development).

Recognizing these limitations, the Policy Committee of the Stand Management Cooperative (SMC) agreed in the fall of 1990 to start a new SMC Modeling Project to address these needs. The first phase was to use existing plot data to develop a new version of the stand-development model ORGANON (Hann et al. 1997) that would be applicable to established managed and unmanaged stands of Douglas-fir and western hemlock throughout the Pacific Northwest. The second was to reestimate the equations in the model as the high-quality modeling data for young managed plots were collected by the SMC Silviculture Project.

ORGANON is classified as a single-tree/distance-independent stand-development model (Munro 1974). Therefore, ORGANON uses a representative sample of trees from the stand as the basis for predicting stand development. Consequently, diameter distribution data are available, as is tree and stand growth information. ORGANON has been developed to be user-friendly and to run on personal computers (PCs). In 1990, there were two versions of ORGANON: SWO-ORGANON for southwest Oregon and NWO-ORGANON for northwest Oregon.

Both versions of ORGANON can project even- and uneven-aged stands composed of pure species or mixed species. For even-aged stands, breast-height stand ages can range from 15 to 120 yr in both versions. Species available in SWO-ORGANON include Douglas-fir, grand fir [*Abies grandis* (Dougl. ex D. Don) Lindl.], white fir [*A. concolor* (Gord. & Glendl.) Lindl. ex Hildebr.], ponderosa pine (*Pinus ponderosa* P. & C. Lawson), sugar pine (*P. lambertiana* Dougl.), incense-cedar [*Calocedrus decurrens* (Torr.) Florin], Pacific madrone (*Arbutus menziesii* Pursh), golden chinkapin [*Chrysolepis chrysophylla* (Dougl. ex Hook.) Helmqv.], California black oak (*Quercus kelloggii* Newberry), and five other minor species, including western hemlock. Species available in NWO-ORGANON include Douglas-fir, grand fir, and three other minor species. The data used to develop both versions came from temporary plots established in 391 stands located in southwest Oregon and 136 stands located on the McDonald-Dunn Research Forests. Therefore, the modeling data sets for both versions included little data from stands under any form of intensive management.

ORGANON contains four sets of equations for predicting 5-yr changes in the attributes[1] of a particular tree species: diameter-growth rate (ΔD), height-growth rate (ΔH), mortality rate, and change in height-to-crown-base (ΔHCB). The mortality rate equations are composed of a single-tree equation for predicting the probability of a tree dying in the next 5 yr (*PM*) and a maximum-size density trajectory equation for capping predicted stand development by increasing mortality if necessary. ΔHCB is predicted from a static height-to-crown-base (*HCB*) equation and constraints placed on it to guarantee proper behavior.

Both ORGANON versions include a simple fertilization response, based on the work of Wang (1990) in southwest Oregon, in their diameter-growth equations. Only 200 lb of nitrogen can be applied, and the response is a function only of time since application. Neither version of ORGANON includes possible thinning response in the equations, other than that predicted by changes in stand density and subsequent changes in the length of tree crowns.

Therefore, the primary goal of the SMC Modeling Project was to develop new *H* from *D* (i.e., *H-D*), *HCB*, ΔD, ΔH, and maximum size-density trajectory equations for Douglas-fir and western hemlock trees growing in southwestern British Columbia, western Washington, and northwestern Oregon. These equations were to include appropriate responses to fertilization and thinning and would form the basis for a new version of ORGANON (i.e.,

[1] Frequently used attributes, variables, and terms are defined and their abbreviations given at first mention in the text. These abbreviations also are summarized in the Appendix.

SMC-ORGANON). Where appropriate, the equations were also to include crown size to allow connections to the wood-quality work of the SMC.

The first phase of the Modeling Project was initiated in January 1991. With the release of the model to cooperators at the end of 1997, the project was terminated and disbanded without starting on the planned second phase. Approximately two-thirds of the project was devoted to acquiring, reformatting, editing, correcting, transforming, summarizing, and storing the data sets and acting on other research requests from the SMC Policy Committee. Data preparation took longer than expected because, first, the project was understaffed as a result of SMC budget restrictions and, second, many of the data sets were received in nonstandard formats. Models were fitted and evaluated during the last third of the project. A regrettable consequence of the termination of the Modeling Project was the loss of the substantial knowledge gained by the project modelers about the strengths and weaknesses of both the existing data sets and the modeling/analytical approaches used or attempted during the project. This loss has greatly hampered or forestalled subsequent efforts to improve the equations.

The objectives of this publication are to describe

- the development of the *HCB*, ΔD, ΔH, *PM*, and maximum size-density trajectory equations created for the SMC version of ORGANON (The *H-D* equations were reported by Hanus et al. 1999.)
- the strengths and weaknesses of the existing data sets used to develop these equations
- the strengths and weaknesses of the modeling and analytical approaches used or attempted during the development of these equations

and to suggest

- alternative approaches that future modeling efforts might explore.

A user's manual (Hann et al. 1997) has been prepared covering the SMC and other versions of ORGANON. It and the ORGANON software are available from the ORGANON web site: www.cof.orst.edu/cof/fr/research/organon.

General Data Description

Data for this study came from existing permanent plots previously established by SMC members in the study area. The following is a summary of the tasks that were undertaken to create the general modeling data set:

(1) Installation and plot-selection criteria were developed.

(2) A format was developed for the raw data requested from the cooperators.

(3) Data-editing programs were developed and documented.

(4) Cooperators were polled about availability of existing data that met the requirements for the modeling work.

(5) Installations were selected for use from the results of the poll.

(6) Requests for data were sent out to cooperators, asking that the data be submitted electronically and in the raw data format developed in task 2.

(7) Data were loaded onto the computer as they were received.

(8) If necessary, the data were reformatted.

(9) Data sets not meeting the criteria developed under task 1 were dropped.

(10) Data were edited.

(11) Data sets with editing problems were either cleaned up by SMC Modeling Project personnel or sent back to cooperators for resolution.

(12) A database management system was developed and fully documented.

(13) Data summarization programs were developed and documented.

(14) The edited data were run through the summarization programs.

(15) If problems arose during data summarization, the data were either cleaned up by project personnel or sent back to cooperators for resolution.

(16) If problems could not be resolved, the data set was dropped.

(17) The edited and summarized data were loaded on the Plot Data Analysis System developed at the Pacific Northwest Research Station of the USDA Forest Service in Olympia, Washington.

The project accumulated a database of 3,345 plots from 371 installations in the study area. Of these, 1,269 plots contained no western hemlock, 389 contained no Douglas-fir, and 1,687 contained both species. The installations ranged from 42.00°N to 50.63°N in latitude and from 120.7°W to 127.68°W in longitude. The data were collected from fixed-area plots averaging 0.17 ac and ranging in size from 0.1 to 1.2 ac. The average breast height age was 27.8 yr and ranged from 3 to 108 yr. Various thinning and fertilization treatments were represented, although most were research, rather than operational, treatments. All plots were measured at least twice. Length of the growth periods between measurements ranged from 1 to 27 yr, with an average of 4.5 yr.

Tree Measurements

Attributes measured included diameter at breast height (*D*) for all sample trees and all measurement times; an indicator of whether the tree had died during the previous growth period for all trees alive at the start of the previous growth period; total height (*H*) for a subsample of the trees measured; and, on some installations, *HCB* for a subsample of the trees measured (usually the same trees that were measured for *H*).

D was measured to the nearest 0.1 in. (0.1 cm in British Columbia) with a diameter tape. *H* was measured by unknown techniques that could have varied from one installation to another. Unmeasured values of *H* were estimated with the plot-level height-diameter fitting procedures of Flewelling and De Jong (1994), combining treatments within an installation whenever possible. *HCB* was measured on a sample of trees; measurement techniques and definitions of the location of *HCB* are unknown. All data measured in metric units were stored in that format and converted to English units during the creation of the modeling data sets.

Calculation of Other Tree and Plot Attributes

Tree and plot attributes previously used to predict ΔD (Hann and Larsen 1991; Zumrawi and Hann 1993) included the site index (*SI*) of the installation, basal area/ac of the plot (*BA*), the *D* and crown ratio (*CR*) of the tree, and the basal area/ac in trees with *D* larger than that of the subject tree on the plot (*BAL*). Attributes previously used to predict ΔH (Hann and Ritchie 1988) included *SI* of the installation, *H* and *CR* of the subject tree, and the percent crown closure of the plot at the tip of the subject tree (*CCH*). Attributes previously used to predict *PM* (Hann and Wang 1990) included *SI* of the installation, *BA* and number of trees/ac (*N*) of the plot, and the *D*, *CR*, and *BAL* of the tree. Finally, the attributes previously used to predict *HCB* (Ritchie and Hann 1987; Zumrawi and Hann 1989) included the *SI* of the installation, the *BA* of the plot, the *H* and *D* of the tree, and the crown competition factor in trees with *D* larger than that of the subject tree (*CCFL*).

For those trees with the appropriate measurements, crown length (*CL*) was calculated by subtracting *HCB* from *H*. *CR* was then computed by dividing *CL* by *H*. For those trees without the appropriate measurements, *HCB* was calculated from the equations reported in this study.

The expansion factor (*EXPAN*) for each sample tree (the number of trees/ac that each sample tree represents) was calculated by taking the reciprocal of the plot area in acres.

SI values were determined for the majority (by basal area) target species (i.e., Douglas-fir and western hemlock) on the installation by computing breast height age and top height (*H40*) for the 40 largest-diameter trees/ac of the majority target species across all of the untreated plots on the installation. These two attributes were calculated for each measurement and remeasurement on an installation; the resulting pair of values with a breast-height age closest to the 50-yr base age of the *SI* equations was then used to determine *SI*. Douglas-fir *SI* (SI_{DF}) was calculated by solving Bruce's (1981) dominant height equations for *SI*; western hemlock *SI* (SI_{WH}) was calculated from *SI* equations of Bonner et al. (1995).

ORGANON projects stand development over a 5-yr growth period. The approach used to model ΔD and ΔH requires exactly 5-yr data, whereas the approach taken to model mortality rate allows the use of data with variable lengths of growth period. The following approach was used to calculate exact 5-yr growth periods for installations in which the total duration of measurements equaled or exceeded 5 yr:

(1) Starting with the first measurement, lengths of growth periods were cumulated until a total of 5 yr was met or exceeded.

(2) If a 5-yr growth period was exceeded by no more than 2 yr, linear interpolation was applied to the measured changes in *D* and *H* during the last growth period in the cumulation. The appropriate fractional value of these measured changes was added to the values at the start of the of the last growth period in order to calculate *D* and *H* at the end of a 5-yr growth period. If, for example, *D* for a tree was measured every 2 yr over 6 yr, with resulting values of 6.0, 6.6, 7.1, and 7.5 in., *D* at the end of the 5-yr growth period would be calculated as 7.3 in. (i.e., 7.1 + (7.5 - 7.1)/2) and the resulting 5-yr ΔD growth rate would be 1.3 in. (i.e., 7.3 - 6.0).

(3) The process then proceeded to the next measurement (i.e., all 5-yr measurement periods started with actual measurements, rather than interpolations), and steps 1 and 2 were repeated until either there were no additional growth periods available or the cumulation for the last period was less than 5 yr.

(4) For SMC installations in which the total duration of measurements was only 4 yr, linear extrapolation was used to calculate *D* and *H* at the end of the 5-yr growth period by multiplying the measured 4-yr changes by 1.25 and adding these expanded increments to the *D* and *H* at the start of the growth period. Cumulated growth periods <4 yr were discarded.

BA, *BAL*, *N*, *CCFL*, and *CCH* in living trees were calculated for each plot on each installation. For the diameter-growth-rate, height-growth-rate and mortality-rate equations, the attributes appropriate for each equation were computed from *D* and *H* at the start of the growth period. For the static *HCB* equations, the attributes appropriate for that equation were computed at each measurement.

BA was calculated by squaring *D* for each sample tree, multiplying it by 0.005454154 and the tree's *EXPAN*, and then summing for all sample trees on the plot. *BAL* was calculated by summing the same values for all sample trees on the plot with *D* larger than that of the subject tree. *N* was calculated by summing *EXPAN* for all sample trees on the plot. *CCFL* was calculated by squaring maximum crown width for each tree, multiplying it by 0.001803 and the tree's *EXPAN*, and then summing the values for all sample trees on the plot with *D* larger than that of the subject tree. The maximum crown width of a tree was estimated by the equations of Paine and Hann (1982), with the equations of Smith (1966) being used for species not found in Paine and Hann (1982).

To calculate *CCH* of a particular tree, *H* of that tree was used to define a reference height (*RH*). Crown widths (*CW*) at *RH* for all other trees on the plot were estimated with the equations described in Hann (1999) and Hann and Hanus (2001). *CW* for each tree was converted to crown area (*CA*) by the formula for the area of a circle. The *CA*s were multiplied by their *EXPAN*s and then summed across all sample trees on the plot and expressed as a percentage of acreage covered. This procedure was repeated for all trees on the plot.

Several attributes characterizing the thinning and the fertilization treatments were also calculated. Attributes used to characterize thinning included (1) the number of thinnings (*nt*) that had occurred at or before either the start of the growth period (ΔD, ΔH, and *PM* equations) or the current measurement (*HCB* equations); (2) the basal area per ac removed in each thinning (BAR_i, $i = 1,\ldots,nt$, with 1 indexing the most recent thinning); (3) the number of trees/ac removed in each thinning (NR_i, $i = 1,\ldots,nt$, with 1 indexing the most recent thinning); (4) the number of growing seasons, in years, since the i^{th} thinning (YT_i, $i = 1,\ldots,nt$, with 1 indexing the most recent thinning); (5) the basal area/ac on the plot before the most recent thinning (*BABT*); and (6) the number of trees/ac on the plot before the most recent thinning (*NBT*). Attributes used to characterize nitrogen fertilization included (1) the number of applications of nitrogen (*nf*) that had occurred at or before either the start of the growth period (for the ΔD, ΔH, and *PM* equations) or the current measurement (for *HCB* equations); (2) the number of lbs/ac of nitrogen applied in each fertilization (PN_i, $i = 1,\ldots,nf$, with 1 indexing the most recent fertilization); and (3) the number of growing seasons, in years, since the i^{th} fertilization (YF_i, $i = 1,\ldots,nf$, with 1 indexing the most recent fertilization). Installations without an unthinned and unfertilized control plot(s) were not used in this study.

Evaluation of the Data

The resulting data sets gathered for Douglas-fir and western hemlock were evaluated for their adequacy in developing the ΔD, ΔH, PM, and HCB equations in ORGANON. Of particular interest was the adequacy of trees with measurements of CR, an important variable in the core equations of ORGANON. This evaluation indicated that the number of tree observations with measurements of H, ΔH, and CR was small. If the measurements were taken on the plots, they were always subsamples of the trees found on the plot. Hs were not always measured on the same tree over time, and they were often concentrated in dominant trees on the plot. Measurements of CR were restricted to those trees with at least one measurement of H. Subsampling was particularly severe in the fertilization data sets from the Regional Forest Nutrition Research Project (RFNRP), in which H measurements were restricted to four dominant trees on each 0.1-ac plot and there were no measurements of CR. No CRs were measured on the other large fertilization data set made available by the British Columbia Ministry of Forests. These data problems largely dictated the analytical approaches taken to develop the four equations.

We also had problems with data from SMC Type 1 installations. These installations were established in young plantations (most with breast height ages <10 yr) or recently respaced natural stands of homogeneous stocking. In these stands, the ΔH rates are much greater than expected from the dominant height growth equations of Bruce (1981) and Bonner et al. (1995). As a result, predicting SI from the Bruce (1981) and Bonner et al. (1995) equations resulted in greatly inflated values for the SMC Type 1 installations. We tried unsuccessfully to derive a "fix" for this problem; therefore, we decided not to use the SMC Type 1 installations in our analyses.

Finally, we occasionally encountered installations or plots for which the documentation, measurement, or both, of initial conditions, past treatments, or measurement techniques was inadequate. For installations without information on the trees removed at the initial spacing treatments, we estimated the initial conditions from data from the control plots, where possible. In some cases, we rejected data from the early measurements of a plot because of the presence of a large number of unmeasured small trees (as evidenced by later ingrowth). Where the problems could not be alleviated, the data were eliminated from further analysis.

Development of Equations

In developing the following equations, we chose to (1) ignore possible serial correlation between repeat measurements, (2) ignore possible lack of independence between trees within a plot, and (3) not report indices of fit for the equations.

Serial correlation can bias estimates of the variances of the parameters and produce inefficient estimates of the parameters themselves; however, the parameters are unbiased (Kmenta 1986). Serial correlation can arise from the use of "time-series data" (repeated measurements of the same tree or plot), rather than "cross-sectional data" (single measurements of many independent trees or plots) (Kmenta 1986). Our data sets are an example of "pooled time-series and cross-sectional data," with many independent plots and relatively short measurement sequences. The use of pooled time-series and cross-sectional data should lessen the potential impact of serial correlation. Furthermore, our ΔD, ΔH, and PM equations use a 5-yr growth period, and Gertner (1985) has found that serial correlation decreases with length of growth period, being almost negligible for a 5-yr period.

The plots used in a growth-and-yield study are considered independent of each other because they have usually been chosen randomly from the population of all possible plots. The trees within the plot, however, are not independent of each other because they were not chosen randomly from the population of all possible trees (Green et al. 1994). This lack of independence can result from spatial correlation between the trees on a plot, which can result in the individual trees impacting each other. This problem is common to all single-tree types of growth-and-yield models and has usually been ignored by their developers. As with serial correlation, the lack of independence of trees on a plot can bias estimates of the variances of the parameters and cause inefficient estimates of the parameters themselves; however, the parameters are still unbiased. We believe the negative impact of the lack of independence between trees can be minimized by incorporating appropriate variables into the prediction equations that characterize the interaction among the trees on the plot. In support of our belief, Reynolds et al. (1988) examined the impact of correlated diameters on the ability to fit diameter distributions to the data and concluded that the correlations among trees were not a severe problem.

The *HCB*, ΔD, and ΔH equations all use weighted regression to homogenize the variance. As a result, the usual indices of fit can be misleading. Furthermore, the *PM* equations were fit by using a nonlinear logistic statistical package that presented no truly meaningful measures of fit (Hamilton 1974). We therefore decided not to report measures of fit for the equations.

Height-to-Crown-Base (HCB)

Data Description

Data for all Douglas-fir and western hemlock trees with a measurement of *HCB* were extracted from the data base. The resulting data were divided into the following five groups for each species:

"untreated" data, consisting of all trees with actual *HCB* measurements from (1) untreated control plots, (2) plots that had been just thinned (i.e., $YT_1 = 0$) and for which *BABT* and *CCFL* before thinning were known, and (3) all measurements from plots that had been thinned more than 20 yr ago. Past experience with modeling *HCB* (Ritchie and Hann 1987; Zumrawi and Hann 1989; Hanus et al. 2000) indicated that the impact of thinning on crown length (and associated *HCBs*) could last for 20 years. The resulting data sets, including variables used in the final *HCB* equations, are described in Table 1.

Table 1. Description of the height-to-crown-base (*HCB*) data sets for Douglas-fir and western hemlock trees on untreated plots. The means were computed from the number of observations (NOb) reported for each variable.

	Douglas-fir		Western hemlock	
Variable (units)	Mean	Range	Mean	Range
Individual tree	*NOb = 11,746*		*NOb = 2,381*	
HCB (ft)	30.2	3.0–136.0	11.4	0.3–86.6
CR	0.56	0.01–0.94	0.74	0.11–0.99
H (ft)	64.9	8.0–188.0	36.0	6.6–128.6
D (in.)	9.4	0.6–46.0	4.8	0.4–15.5
CCFL (ft^2/ac)	133.5	0.0–532.7	142.0	0.0–457.6
D/H	0.14	0.05–0.27	0.13	0.06–0.30
Individual plot	*NOb = 686*		*NOb = 47*	
BH AGE	31.2	13.0–108.0	30.8	8.2–108.0
BA (ft^2/ac)	140.8	36.6–406.1	201.6	21.8–352.2
Installation	*NOb = 32*		*NOb = 10*	
SI_{SP}	118.5	77.6–155.0	102.1	82.1–123.6

"single thinning" data, consisting of all trees with actual *HCB* measurements from plots that had been thinned only once. Included in these data were plots that had been just thinned (i.e., $YT_1 = 0$) but, in this use of the data, *BA* and *CCFL* after thinning were used in the data set. The resulting data sets, including variables used in the final *HCB* equations, are described in Table 2.

Table 2. Description of the height-to-crown-base (*HCB*) data sets for Douglas-fir and western hemlock trees receiving a single thinning. The means were computed from the number of observations reported for each variable.

Variable	Douglas-fir		Western hemlock	
	Mean	Range	Mean	Range
Individual tree	*NOb* = 4,073		*NOb* = 573	
HCB (ft)	26.2	2.0–150.0	7.7	0.3–49.5
CR	0.59	0.10–0.91	0.83	0.29–0.99
H (ft)	57.6	14.1–205.0	37.9	16.0–94.2
D (in.)	8.3	1.3–44.1	6.5	1.8–15.4
CCFL (ft^2/ac)	111.6	0.0–431.5	78.1	0.0–302.7
D/H	0.14	0.07–0.29	0.17	0.09–0.30
Individual plot	*NOb* = 179		*NOb* = 14	
BH AGE	33.6	13.0–108.0	27.2	8.2–108.0
BA (ft^2/ac)	127.3	25.0–412.8	121.9	37.5–254.2
nt	1.0	1.0–1.0	1.0	1.0–1.0
YT_1	12.3	0.0–57.0	10.1	2.0–57.0
$PREM_1$	0.496	0.015–0.922	0.474	0.015–0.691
Installation	*NOb* = 19		*NOb* = 4	
SI_{SP}	113.3	77.6–142.0	100.6	83.2–121.0

"single fertilization" data, consisting of all trees with actual *HCB* measurements from plots that had been fertilized once with ≤450 PN (lb nitrogen)/ac. The resulting data sets, including variables used in the final *HCB* equations, are described in Table 3.

Table 3. Description of the height-to-crown-base (*HCB*) data sets for Douglas-fir and western hemlock trees receiving a single fertilization. The means were computed from the number of observations reported for each variable.

Variable	Douglas-fir		Western hemlock	
	Mean	Range	Mean	Range
Individual tree	*NOb* = 3,524		*NOb* = 17	
HCB (ft)	16.1	3.9–51.0	24.5	5.2–48.0
CR	0.59	0.04–0.88	0.56	0.35–0.89
H (ft)	38.3	7.9–88.5	54.2	11.2–81.0
D (in.)	4.2	1.0–17.0	7.4	1.5–13.1
CCFL (ft^2/ac)	218.2	0.0–472.2	192.6	15.0–351.3
D/H	0.11	0.06–0.19	0.13	0.08–0.19
Individual plot	*NOb* = 49		*NOb* = 7	
BH AGE	26.8	16.0–43.0	22.3	16.0–43.0
BA (ft^2/ac)	172.9	81.1–228.1	129.5	81.1–219.8
nf	1.0	1.0–1.0	1.0	1.0–1.0
YF_1	10.1	0.0–18.0	3.6	0.0–13.0
PN_1	297.8	200.0–400.0	285.5	200.0–400.0
Installation	*NOb* = 3		*NOb* = 3	
SI_{SP}	92.3	78.1–100.7	87.2	71.6–98.0

"multiple thinning" data, consisting of all trees with actual *HCB* measurements from plots that had been thinned more than once. The resulting data sets, including variables used in the final *HCB* equations, are described in Table 4.

Table 4. Description of the height-to-crown-base (*HCB*) data sets for Douglas-fir and western hemlock trees receiving multiple thinnings. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 10,112*		*NOb = 319*	
HCB (ft)	47.6	6.0–145.0	48.1	4.0–84.6
CR	0.50	0.01–0.90	0.51	0.17–0.90
H (ft)	92.1	18.0–203.0	48.1	14.0–132.5
D (in.)	14.1	2.7–53.7	12.7	1.7–24.3
CCFL (ft^2/ac)	105.4	0.0–461.2	155.2	0.0–335.8
D/H	0.15	0.06–0.31	0.13	0.08–0.21
Individual plot	*NOb = 888*		*NOb = 64*	
BH AGE	39.0	16.0–108.0	48.2	35.0–108.0
BA (ft^2/ac)	151.8	62.3–354.9	183.1	83.7–300.3
nt	4.5	2.0–7.0	5.5	3.0–6.0
YT_1	6.0	0.0–36.0	12.1	6.9–32.0
$PREM_1$	0.143	0.005–0.608	0.136	0.005–0.513
Installation	*NOb = 17*		*NOb = 5*	
SI_{SP}	124.8	85.8–140.5	102.4	82.1–123.6

"single thinning and single fertilization" data, consisting of all trees with actual *HCB* measurements from plots that had been thinned only once and fertilized only once. The thinning and fertilization did not have to occur at the same time. The resulting data sets, including variables used in the final *HCB* equations, are described in Table 5.

Table 5. Description of the height-to-crown-base (*HCB*) data sets for Douglas-fir and western hemlock trees receiving a single thinning and a single fertilization. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 3,403*		*NOb = 19*	
HCB (ft)	16.1	2.9–50.0	31.9	7.5–42.0
CR	0.66	0.21–0.89	0.57	0.47–0.88
H (ft)	45.9	12.4–98.3	72.9	28.9–88.4
D (in.)	5.7	1.3–16.2	10.1	2.8–14.0
CCFL (ft^2/ac)	148.4	0.0–391.6	173.4	31.9–304.1
D/H	0.12	0.07–0.22	0.14	0.10–0.17
Individual plot	*NOb = 99*		*NOb = 4*	
BH AGE	27.0	16.0–43.0	31.5	17.0–43.0
BA (ft^2/ac)	139.9	75.7–230.7	182.3	127.8–230.7
nt	1.0	1.0–1.0	1.0	1.0–1.0
YT_1	10.0	0.0–18.0	6.0	0.0–9.0
$PREM_1$	0.459	0.013–0.683	0.237	0.029–0.433
nf	1.0	1.0–1.0	1.0	1.0–1.0
YF_1	10.1	0.0–18.0	8.0	0.0–13.0
PN_1	296.8	200.0–400.0	300.0	200.0–400.0
Installation	*NOb = 3*		*NOb = 2*	
SI_{SP}	92.3	78.1–100.7	84.8	71.6–98.0

Data Analysis and Results

Equation for Untreated Plots

The "untreated" data (Table 1) were used to estimate the parameters of the *HCB* equation used by Ritchie and Hann (1987) and Hanus et al. (2000):

$$HCB = \frac{H}{1.0 + e^{x}} \quad [1]$$

$$x = d_0 + d_1 H + d_2 CCFL + d_3 \ln(BA) + d_4 D/H + d_5 SI_{SP}$$

where

SI_{SP} = site index for species "SP"

SP = DF for Douglas-fir

= WH for western hemlock

Ritchie and Hann (1987), Zumrawi and Hann (1989), and Hanus et al. (2000) all found that the variance of the residuals in *HCB* increased with *H*. Thus, they used weighted regression with a weight of $(1.0/H)^2$ when estimating the parameters of their equation. Therefore, weighted nonlinear regression, with a weight of $(1.0/H)^2$, was also used to estimate the parameters in this study. Each parameter was then tested for statistical significance from 0 ($P = 0.05$) with a *t*-test. If not significant, it was set to 0 and the remaining parameters were reestimated by using weighted nonlinear regression. The parameter estimates and their standard errors (SE) for Eq. [1] are provided in Table 6.

Table 6. Parameters and asymptotic standard errors for predicting height-to-crown-base (*HCB*) for untreated Douglas-fir and western hemlock, Eq. [1].

Parameter/ Standard error	Douglas-fir	Western hemlock
d_0	3.411317351	8.856979690
SE(d_0)	(0.06287614415)	(0.19321795751)
d_1	-0.009947861	-0.004799358
SE(d_1)	(0.00025401814)	(0.00078728937)
d_2	-0.001906272	-0.000842256
SE(d_2)	(0.00007665450)	(0.00014913736)
d_3	-0.656269205	-1.661329351
SE(d_3)	(0.01380472545)	(0.04133363527)
d_4	4.520522655	5.485552579
SE(d_4)	(0.22927864423)	(0.48709621978)
d_5	0.002595706	0.0
SE(d_5)	(0.00021596669)	(NA)

NA: Not applicable.

Equation for Thinned Plots

Because *BA* and *CCFL* can be modified by thinning, predictions of *HCB* after thinning from Eq. [1] can be biased because they will predict an immediate reduction in *HCB*. In order to reduce this bias, the impact of past thinning(s) on predicted *HCB* was modeled as an effect on the *CCFL* and *BA* attributes in the following manner:

$$CCFL_T = CCFL + \sum_{i=1}^{nt} CCFLR_i e^{d_6 YT_i^2}$$

$$BA_T = BA + \sum_{i=1}^{nt} BAR_i e^{d_6 YT_i^2}$$

where $CCFLR_i$ = plot crown competition factor in trees with D >that of the subject tree removed in the i^{th} thinning

These combined attributes were then inserted into the following equation form for estimating HCB for thinned plots:

$$x = d_0 + d_1 H + d_2 CCFL_T + d_3 \ln(BA_T) + d_4 D/H + d_5 SI_{SP} \qquad [2]$$

Parameters d_0 through d_5 were fixed to the values from the "untreated" fit (Table 6), and d_6 was estimated using the "thinned" data sets described in Tables 2 and 4 and weighted nonlinear regression. The thinning parameter estimate and its SE for Eq. [2] are found in Table 7.

Table 7. Parameter and asymptotic standard error for predicting height-to-crown-base (*HCB*) for thinned Douglas-fir and western hemlock, Eq. [2].

Parameter/ Standard error	Douglas-fir	Western hemlock
d_6	-0.0596453035	-0.0043929792
SE(d_6)	(0.00778484588)	(0.00041779171)

Equation for Fertilized Plots

For Douglas-fir, all of the *HCB* data from fertilized plots came from three installations that had received only a single fertilization (Table 3), and most of the data originated on only one of the three installations. The data set for western hemlock (Table 3) was too small to analyze. Therefore, the following analysis was restricted to a single fertilization of Douglas-fir. The purpose of this analysis was to explore the potential impact of fertilization on *HCB*, rather than to develop a fully functioning predictive equation, which would require a more comprehensive modeling data set.

After examining several alternatives, we modeled the impact of a single fertilization on predicted *HCB*, using the following variable:

$$FERT = d_7 (PN_1/800.0)^{d_8} \, YF_1^{1/2} \, e^{d_9 YF_1^2}$$

This variable was then inserted into the following equation form for estimating *HCB* for plots receiving a single fertilization:

$$x = d_0 + d_1 H + d_2 CCFL + d_3 \ln(BA) + d_4 D/H + d_5 SI_{SP} + FERT \qquad [3]$$

Parameters d_0 through d_5 were fixed to the values from the untreated plot equation

Table 8. Parameters and asymptotic standard errors for predicting height-to-crown-base (*HCB*) for a single fertilization of Douglas-fir, Eq. [3].

Parameter/ Standard error	Douglas-fir
d_7	0.3359858889
SE(d_7)	(0.03683906254)
d_8	0.3687658812
SE(d_8)	(0.08608144107)
d_9	-0.0139853300
SE(d_9)	(0.00130116711)

(Table 6), and parameters d_7, d_8 and d_9 were estimated by using the "single fertilization" data set for Douglas-fir described in Table 3 and weighted nonlinear regression. The fertilization parameter estimates and their SEs for Eq. [3] are given in Table 8.

This analysis assumed that parameters d_0 through d_5 for the untreated fit (Table 6) were adequate for characterizing the *HCB* for untreated trees on the three installations with single fertilization data. To check this assumption, Eq. [3] was applied to each tree on the control plots with a measured *HCB*, and the difference between predicted *HCB* and measured *HCB* was then calculated. The mean of the difference for the 1,671 trees on the control plots was 0.79 ft, indicating that the assumption used in the fertilization analysis was reasonable.

Discussion

HCB Eq. [1] for untreated plots and the associated parameter estimates for each species (Table 6) predict an increase in *HCB* with an increase in *H*, *CCFL*, and *BA*, and a decrease in *HCB* with an increase in *D/H* and SI_{SP}. These results are in agreement with those of Ritchie and Hann (1987), Zumrawi and Hann (1989), and Hanus et al. (2000).

The geographic area from which the data used to develop the Zumrawi and Hann (1989) equation for Douglas-fir were collected falls within the geographic area for the SMC study. Therefore, predictions from the Zumrawi and Hann (1989) Douglas-fir equation were compared to predictions from Eq. [1] for Douglas-fir. For the same tree and plot attributes, Eq. [1] predicted shorter *HCB* (and therefore longer *CL*) than the equation from Zumrawi and Hann (1989). Hanus et al. (2000) have shown that, for the same tree and plot attributes, undamaged trees have longer *CLs* than do damaged trees. The Zumrawi and Hann (1989) data set included all trees on the plot and therefore incorporated both damaged and undamaged trees. The data used in this study were subsampled mostly on those trees measured for *H*. We hypothesized that the data used in this analysis were concentrated in undamaged trees, and, therefore, the equations predict longer *CLs* than the overall population would exhibit.

The thinning-effects modifier in Eq. [2] and the associated parameter for each species (Table 7) predict that the *HCB* will be the same immediately before and immediately after thinning. The *HCB* will start to increase as tree and plot attributes develop and as the time since thinning increases.

The fertilization-effects modifiers in Eq. [3] and the associated parameters for Douglas-fir (Table 8) predict that (1) the *HCB* fertilization will be the same immediately before and immediately after fertilization, (2) *HCB* will remain lower (or decline) for the fertilized tree when compared to a tree not fertilized until $YF_I \simeq 4.2$ yr, and (3) after $\simeq 4.2$ yr, the direct impact of fertilization will disappear as YF_I continues to increase. (The indirect impact caused by changing the other tree and plot attributes in Eq. [1] would remain.) The result would be an increase in crown length for a period after fertilizing a tree.

Five-Year Diameter-Growth Rate (ΔD)

Data Description

All ΔD data for Douglas-fir and western hemlock were extracted from the data base and then divided into the following eight groups for each species:

"untreated with CR" data, consisting of all trees with actual CR measurements at the start of the growth period from untreated control plots of at least 0.2 ac. The resulting data sets, including variables used in the final ΔD equations, are described in Table 9.

Table 9. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured crown ratios on untreated plots. The means were computed from the number of observations reported for each variable.

Variable	Douglas-fir Mean	Douglas-fir Range	Western hemlock Mean	Western hemlock Range
Individual tree	*NOb = 4,093*		*NOb = 881*	
ΔD	0.92	-0.1–3.9	1.79	0.0–4.0
D (in.)	8.33	0.6–36.7	3.41	0.1–14.3
CR	0.53	0.05–0.92	0.71	0.11–0.92
BAL	85.6	0.0–365.1	41.5	0.0–280.2
Individual plot	*NOb = 168*		*NOb = 28*	
BA (ft^2/ac)	198.3	24.6–385.1	199.4	11.6–325.3
$BH\ AGE$	34.8	11.0–77.0	28.0	6.2–44.1
Installation	*NOb = 27*		*NOb = 7*	
SI_{SP}	118.9	77.6–155.0	109.9	91.9–123.6

"untreated with predicted CR" data, consisting of all measurements with measured *H*s such that *CR* at the start of the growth period could be predicted from untreated control plots of at least 0.2 ac by using Eq. [1]. The resulting data sets, including variables used in the final ΔD equations, are described in Table 10.

Table 10. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with predicted crown ratios on untreated plots. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 13,149*		*NOb = 1,955*	
ΔD	0.85	-0.1–4.1	0.68	-0.1–3.1
D (in.)	8.97	0.1–39.3	8.70	0.2–24.4
PCR	0.55	0.11–0.99	0.39	0.11–0.98
BAL	74.9	0.0–398.3	130.3	0.0–406.7
Individual plot	*NOb = 1,060*		*NOb = 230*	
BA (ft^2/ac)	182.0	9.1–411.1	238.6	11.6–411.1
BH AGE	34.4	10.0–85.1	36.9	6.2–85.1
Installation	*NOb = 208*		*NOb = 75*	
SI_{SP}	108.7	56.1–156.0	103.1	43.0–131.0

"single thinning" data, consisting of all measurements with actual *CR* measurements at the start of the growth period from plots of at least 0.2 ac that had been thinned only once. The resulting data sets, including variables used in the final ΔD equations, are described in Table 11.

Table 11. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured crown ratios receiving a single thinning. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 2,852*		*NOb = 68*	
ΔD	1.03	-0.1–3.8	3.11	0.9–4.3
D (in.)	7.07	1.0–43.0	3.59	1.5–10.9
CR	0.63	0.01–0.91	0.78	0.46–0.90
BAL	43.1	0.0–279.9	18.1	0.0–127.9
Individual plot	*NOb = 141*		*NOb = 2*	
BA (ft^2/ac)	78.7	15.5–298.1	77.8	20.5–135.0
BH AGE	28.0	11.0–77.0	20.1	6.2–34.0
nt	1.0	1.0–1.0	1.0	1.0–1.0
YT_1	8.3	0.0–33.0	0.0	0.0–0.0
$PREM_1$	0.505	0.043–0.922	0.191	0.069–0.312
Installation	*NOb = 15*		*NOb = 2*	
SI_{SP}	110.0	77.6–142.0	109.5	98.2–121.0

"single fertilization" data, consisting of all measurements from plots that had been fertilized only once. Because of the lack of measured *CR*s on most installations, *CR* values of trees with measured *H*s were predicted from Eq. [1] (PCR_{SMC}). Most of these plots were 0.1 ac. The resulting data sets, including variables used in the final ΔD equations, are described in Table 12.

Table 12. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured heights and predicted crown ratios receiving a single fertilization. The means were computed from the number of observations reported for each variable.

Variable	Douglas-fir Mean	Douglas-fir Range	Western hemlock Mean	Western hemlock Range
Individual tree	*NOb = 11,945*		*NOb = 2,258*	
ΔD	1.05	-0.1–4.5	0.76	-0.1–3.6
D (in.)	8.63	1.0–32.0	9.64	1.8–27.1
PCR	0.57	0.11–0.99	0.37	0.10–0.92
BAL	76.6	0.0–409.7	133.6	0.0–410.8
Individual plot	*NOb = 954*		*NOb = 225*	
BA (ft^2/ac)	187.1	9.6–412.3	267.8	54.9–412.3
BH AGE	34.2	10.6–85.1	39.2	11.6–85.1
nf	1.0	1.0–1.0	1.0	1.0–1.0
YF_1	2.8	0.0–12.0	2.7	0.0–6.1
PN_1	281.4	33.9–803.0	343.5	200.0–803.0
Installation	*NOb = 167*		*NOb = 47*	
SI_{SP}	108.2	56.1–156.0	107.3	43.0–131.0

"single thinning and fertilization" data, consisting of all measurements from plots that had been both thinned and fertilized only once. The thinning and fertilization did not have to occur at the same time. *CR* values of trees with measured *H*s were predicted from Eq. [2]. Most of these plots were 0.1 ac. The resulting data sets, including variables used in the final ΔD equations, are described in Table 13.

Table 13. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured heights and predicted crown ratios receiving a single thinning and a single fertilization. The means were computed from the number of observations reported for each variable.

Variable	Douglas-fir Mean	Douglas-fir Range	Western hemlock Mean	Western hemlock Range
Individual tree	*NOb = 3,724*		*NOb = 36*	
ΔD	1.13	-0.1–3.9	1.35	0.3–2.2
D (in.)	7.76	0.8–36.1	8.29	5.4–10.5
PCR	0.63	0.16–0.99	0.56	0.47–0.88
BAL	69.6	0.0–342.5	90.0	15.7–143.4
Individual plot	*NOb = 227*		*NOb = 4*	
BA (ft^2/ac)	157.7	11.2–347.6	144.6	143.5–144.9
BH AGE	40.5	10.6–74.2	34.0	34.0–34.0
nt	1.0	1.0–1.0	1.0	1.0–1.0
YT_1	4.6	0.0–12.0	0.0	0.0–0.0
$PREM_1$	0.332	0.005–0.731	0.028	0.005–0.053
nf	1.0	1.0–1.0	1.0	1.0–1.0
YF_1	4.8	0.0–12.0	4.0	4.0–4.0
PN_1	292.3	200.0–602.2	200.0	200.0–200.0
Installation	*NOb = 31*		*NOb = 1*	
SI_{SP}	101.1	63.0–156.0	98.0	98.0–98.0

"multiple thinning" data, consisting of all measurements with actual *CR* measurements at the start of the growth period from plots of at least 0.2 ac that had been thinned more than once. The resulting data sets, including variables used in the final ΔD equations, are described in Table 14.

Table 14. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured crown ratios receiving multiple thinnings. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 6,290*		*NOb = 260*	
ΔD	1.33	-0.1–4.7	0.86	0.0–2.7
D (in.)	13.45	3.0–37.5	12.17	5.9–23.0
CR	0.51	0.06–0.90	0.54	0.17–0.89
BAL	72.4	0.0–279.6	109.1	10.3–219.9
Individual plot	*NOb = 489*		*NOb = 54*	
BA (ft^2/ac)	135.1	42.8–312.1	147.5	67.5–262.6
BH AGE	38.5	16.0–77.0	38.6	26.0–44.1
nt	4.5	2.0–7.0	5.9	4.0–6.0
YT_1	4.4	0.0–31.0	4.5	0.0–9.0
$PREM_1$	0.159	0.008–0.646	0.117	0.026–0.211
Installation	*NOb = 12*		*NOb = 2*	
SI_{SP}	123.4	85.8–137.9	123.3	123.0–123.6

"multiple fertilization" data, consisting of all measurements from plots that had been fertilized more than once. *CR* values of trees with measured *H*s were predicted from Eq. [1]. The resulting data consisted of 19 tree-level measurements from one installation for Douglas-fir and 44 tree-level measurements from one installation for western hemlock. All other data from these plots were rejected for one of two reasons: (1) many of the plots had been initially thinned before the first measurement, or (2) the multiple fertilization treatments were on a 2- or 4-yr measurement cycle, making it impossible to create a 5-yr growth period. The resulting data sets, including variables used in the final ΔD equations, are described in Table 15.

Table 15. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured heights and predicted crown ratios receiving multiple fertilizations. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 19*		*NOb = 44*	
ΔD	0.90	0.0–1.7	0.93	0.0–2.1
D (in.)	15.95	9.8–22.7	8.20	3.6–12.4
PCR	0.38	0.27–0.45	0.42	0.28–0.53
BAL	67.2	0.0–191.5	82.8	0.0–240.4
Individual plot	*NOb = 3*	*NOb = 6*		
BA (ft^2/ac)	247.4	230.7–270.1	246.4	224.2–264.6
BH AGE	47.0	47.0–47.0	27.9	27.9–27.9
nf	2.7	2.0–3.0	3.0	3.0–3.0
YF_1	0.0	0.0–0.0	0.0	0.0–0.0
PN_1	200.0	200.0–200.0	300.0	200.0–400.0
Installation	*NOb = 1*	*NOb = 1*		
SI_{SP}	116.0	116.0–116.0	100.0	100.0–100.0

"multiple thinning and fertilization" data, consisting of all measurements from plots that had been both thinned and fertilized more than once. The thinning and fertilization did not have to occur at the same time. *CR* values of trees with measured *Hs* were predicted from Eq. [2] (PCR_{SMC}). The resulting data consisted of 25 tree-level measurements from three installations for Douglas-fir and 70 tree-level measurements from one installation for western hemlock. All other data were rejected for one of the two reasons given in the preceding paragraph. The resulting data sets, including variables used in the final ΔD equations, are described in Table 16.

Table 16. Description of the 5-yr diameter-growth-rate (ΔD) data sets for Douglas-fir and western hemlock trees with measured heights and predicted crown ratios receiving multiple thinnings and fertilizations. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb* = 25		*NOb* = 70	
ΔD	1.60	0.2–2.8	1.06	0.2–2.0
D (in.)	10.07	6.3–15.3	8.32	3.9–11.6
CR or *PCR*	0.63	0.50–0.71	0.52	0.32–0.67
BAL	60.0	0.0–155.6	70.3	0.0–225.6
Individual plot	*NOb* = 3		*NOb* = 10	
BA (ft^2/ac)	140.1	123.8–158.6	187.0	145.6–226.4
BH AGE	20.8	20.5–21.0	27.9	27.9–27.9
nt	2.0	2.0–2.0	3.0	3.0–3.0
YT_1	0.0	0.0–0.0	0.0	0.0–0.0
$PREM_1$	0.157	0.071–0.23	0.141	0.073–0.218
nf	1.0	1.0–1.0	3.0	3.0–3.0
YF_1	8.0	8.0–8.0	0.0	0.0–0.0
PN_1	200.0	200.0–200.0	320.0	200.0–400.0
Installation	*NOb* = 3		*NOb* = 1	
SI_{SP}	129.0	107.0–156.0	100.0	100.0–100.0

Data Analysis and Results

The general approach of Wang (1990) was taken to model ΔD. First, an equation for predicting the growth rate of untreated trees was developed. Multiplicative modifiers to the equation for untreated plots that characterized the effect of thinning and fertilization on ΔD were then developed:

$$\Delta D = \Delta D_c \cdot TR_{\Delta D} \cdot FR_{\Delta D}$$

where

ΔD_c = predicted 5-yr ΔD of an untreated tree

$TR_{\Delta D}$ = predicted thinning response of 5-yr ΔD

$FR_{\Delta D}$ = predicted fertilization response of 5-yr ΔD

Equation for Untreated Plots

The following equation form was fit to the "control data with CR" data sets for both species by weighted nonlinear regression:

$$\Delta D_c = e^{a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5 + a_6 X_6} \quad [4]$$

where

$X_1 = \ln(D + 1)$

$X_2 = D^2$

$X_3 = \ln(SI_{SP} - 4.5)$

$X_4 = \ln[(CR + 0.2)/1.2]$

$X_5 = BAL^2/\ln(D + 5.0)$

$X_6 = BA^{1/2}$

This equation form has been used previously to model ΔD of predominantly untreated stands in both SWO-ORGANON (Hann and Larsen 1991) and NWO-ORGANON (Zumrawi and Hann 1993). As in the previous work, weighting by the reciprocal of predicted ΔD was used to homogenize the variance.

Examination of the resulting parameters indicated that predictions from the equation were not reasonable for either species, based on our previous experiences with modeling ΔD (e.g., Hann and Larsen 1991, Zumrawi and Hann 1993). For Douglas-fir, the predicted maximum ΔDs were judged to be too high; for western hemlock, the parameter on SI_{WH} was insignificant ($P = 0.05$). We hypothesized that these problems were caused by the small data sets available with measured CR. To expand the modeling data, the "untreated with predicted CR" data were combined with the "untreated with CR" data and the following equation was fit to the combined data by weighted nonlinear regression:

$$\Delta D_c = e^{a_0 + a_1 X_1 + a_3 X_3 + a_4 + a_5 + a_6 X_6 + a_7 (1.0 - I_{CR}) X_7} \quad [5]$$

where

I_{CR} = 1.0 if CR was measured

= 0.0 if CR was predicted

$X_7 = \ln[(PCR_{SMC} + 0.2)/1.2]$

The resulting parameters for western hemlock appeared to be reasonably well behaved, and the parameter on SI_{WH} was significantly different from 0 ($P = 0.05$). The parameters for Douglas-fir were still judged to be unreasonable, particularly in the effect of D on predicted ΔD, as judged by the previous work of Hann and Larsen (1991) and Zumrawi and Hann (1993). Therefore, the D-related parameters (i.e., a_1 and a_2) were fixed at the values from Hann and Larsen (1991) and the values of Zumrawi and Hann (1993), and the remaining parameters of Eq. [5] were fit to the combined data set by weighted nonlinear regression. Both fits produced parameters that were judged to be reasonable in behavior. Because the fit with the a_1 and a_2 parameters fixed at the values from Hann and Larsen (1991) produced a smaller mean square error (MSE) than the fit with the values from Zumrawi and Hann (1993), the former were chosen as the final values for predicting untreated ΔD of Douglas-fir. The species-specific parameter estimates and their SEs for Eq. [5] are found in Table 17.

Table 17. Parameters and asymptotic standard errors for predicting the 5-yr diameter-growth rate (ΔD_5) of untreated Douglas-fir and western hemlock, Eq. [5].

Parameter/ Standard error	Douglas-fir	Western hemlock
a_0	-5.252294703	-6.163271737
SE(a_0)	(0.06200533381)	(0.35373005655)
a_1	0.401284000	0.349727789
SE(a_1)	(NA)	(0.02607114536)
a_2	-0.000444053	-0.002303713
SE(a_2)	(NA)	(0.00019786198)
a_3	1.142705108	1.395206036
SE(a_3)	(0.01398773570)	(0.07196248990)
a_4	1.191474443	1.000278663
SE(a_4)	(0.03475820691)	(0.05920037290)
a_5	-0.000048600	-0.000049948
SE(a_5)	(0.00000102903)	(0.00000192082)
a_6	-0.016648482	0.0
SE(a_6)	(0.00223926862)	(NA)
a_7	1.038401774	1.299605189
SE(a_7)	(0.03374634849)	(0.06444912821)

NA: not applicable.

Equation for a Single Thinning

The effect of a single thinning on ΔD was modeled as a multiplier on the untreated-plot equations. The untreated-plot equation was first calibrated to the control plot(s) found on each installation that contained plots with a single thinning in order to reduce variation caused by between-plot differences in the ΔD relationship (Hann and Hanus 2002). This calibration was done by regressing each plot's control-plot data with a measured CR on the predicted values from Eq. [5] for untreated plots by weighted linear regression:

$$\Delta D_{c,i,j} = k_{ST,j}\ Pred\Delta D_{c,i,j}$$

where

$\Delta D_{C,i,j}$ = measured ΔD for trees with measured CR on the untreated plots in the j^{th} installation that included single thinning data with measured CRs

$Pred\Delta D_{C,i,j}$ = predicted ΔD from Eq. [5] for the trees with measured CR from the untreated plots on the j^{th} installation that included single thinning data with measured CRs

$k_{ST,j}$ = untreated tree calibration for all of the untreated plots on the j^{th} installation that included single thinning data with measurements of CRs

i = $1,\ldots,n_j$

n_j = the number of sample trees with measured CRs from all of the untreated plots on the j^{th} installation that included single thinning data

The values of $k_{ST,j}$ were estimated by using weighted linear regression and a weight of $(Pred\Delta D_{C,i,j})^{-1}$. For each installation and 5-yr growth period, the ΔD for each tree on the thinned plots with a measured CR was then predicted by the calibrated untreated plot equation, and the ratio of the tree's actual ΔD to the calibrated predicted ΔD was calculated:

$$STR_{\Delta D,i,j} = \Delta D_{T,i,j} / (k_{ST,j}\ Pred\Delta D_{C,i,j})$$

This ratio, therefore, estimates any additional effect of a single thinning on ΔD not reflected in the values of the tree and plot attributes incorporated in the equation for untreated plots.

The $STR_{\Delta D,i,j}$ data were combined across all installations ($STR_{\Delta D}$). Graphs of $STR_{\Delta D}$ across BAR_T, $BABT$, the proportion of $BABT$ removed in the thinning ($PREM_{\Delta D}$), the ratio of the quadratic mean diameter of the trees cut in the thinning to the quadratic mean diameter of all trees before thinning (QMD_T/QMD_B), YT_T, CR, BAL, and BA were examined for Douglas-fir, the largest data set. These graphs indicated that $STR_{\Delta D}$

increased with both $PREM_{\Delta D}$ and QMD_T/QMD_B and decreased with YT_I. No effect of D, CR, BAL, or BA could be detected. After examining many alternative formulations with these attributes, we concluded that the very high correlation between $PREM_{\Delta D}$ and QMD_T/QMD_B adversely affected the ability to estimate the parameters of an equation form incorporating them both. Therefore, the following equation form was judged best for characterizing the impact of a single thinning on the ΔD of Douglas-fir:

$$STR_{\Delta D} = 1.0 + a_8(PREM_{\Delta D})e^{a_9 YT_I} \quad [6]$$

This equation was fit to the combined Douglas-fir $STR_{\Delta D}$ data (Table 11) by nonlinear regression. A graph of the resulting residuals indicated homogeneous variance, so weighting was deemed unnecessary.

The western hemlock data with measured CRs available to fit Eq. [6] comprised just 68 trees from 2 installations (Table 11); this was judged too small a data set. As an alternative, western hemlock trees with PCR_{SMC} on single thinned plots were extracted from the data base (Table 18). Because PCR_{SMC} was used, we felt that only the first measurement after thinning (i.e., YT_I at the start of the growth period was <1) was legitimate for this analysis because of the impact of thinning on CR.

Equation [6] was then fit to the alternative western hemlock data set with nonlinear regression. Because the data were restricted to $YT_I < 1$, the a_9 parameter for western hemlock was set to the value estimated for Douglas-fir. The species-specific parameter estimates and their SEs for Eq. [6] are given in Table 19.

Table 18. Description of the 5-yr diameter-growth-rate (ΔD) data sets for western hemlock trees with predicted crown ratios receiving a single thinning in which $YT_1 < 1$. The means were computed from the number of observations reported for each variable.

Variable	Mean	Range
Individual tree	*NOb = 221*	
ΔD	1.89	0.1–4.3
D	5.76	1.5–14.9
PCR	0.72	0.33–0.91
BAL	40.3	0.0–171.3
Individual plot	*NOb = 20*	
BA (ft²/ac)	117.3	29.5–246.3
BH AGE	25.9	6.2–51.0
nt	1.0	1.0–1.0
YT_I	0.0	0.0–0.0
$PREM_I$	0.338	0.028–0.802
Installation	*NOb = 8*	
SI_{WH}	103.2	83.2–122.0

Table 19. Parameters and asymptotic standard errors for predicting the response of 5-yr diameter-growth rate (ΔD) to a single thinning in Douglas-fir and western hemlock, Eq. [6].

Parameter/ Standard error	Douglas-fir	Western hemlock
a_8	0.7935257265	0.723095045
SE(a_8)	(0.02219882797)	(NA)
a_9	-0.1257128869	-0.1257128869
SE(a_9)	(0.01127198497)	(NA)

NA: Not applicable.

Equation for a Single Fertilization

To evaluate how fertilizer response changes over time, Miller and Tarrant (1983), Auchmoody (1985), and Opalach and Heath (1988) have partitioned long-term fertilizer response into direct and indirect effects. Opalach and Heath (1988) defined the direct effect as "...that part of the response due to improved nutrition...", and the indirect effect as "...the remaining portion of the response due to altered stocking brought on by fertilizer in previous growing seasons". In general, the direct effect is the response that modelers attempt to estimate in the development of fertilizer response equations for growth-and-yield models (Wang 1990).

The effect of a single fertilization on ΔD was modeled as a multiplier on the untreated-plot equations. The untreated-plot equation was first calibrated to the control plot(s) on the installation in order to reduce variation caused by between-plot differences in the ΔD relationship (Hann and Hanus 2002). This calibration was done by regressing each plot's control-plot data on the predicted values from the untreated equation with weighted linear regression:

$$\Delta D_{C,i,j} = k_{SFj}\ Pred\Delta D_{C,i,j}$$

where

$\Delta D_{C,i,j}$ = measured ΔD for trees with predicted CR on all of the untreated plots in the j^{th} installation that included single-fertilization data

$Pred\Delta D_{C,i,j}$ = predicted ΔD from Eq. [5] for the trees with predicted CR from all of the untreated plots on the j^{th} installation that included single-fertilization data

k_{SFj} = untreated-tree calibration for all of the untreated plots on the j^{th} installation that included single-fertilization data

i = $1,\ldots,n_j$

n_j = the number of sample trees with predicted CR on all of the untreated plots on the j^{th} installation that included single-fertilization data

The values of k_{SFj} were estimated by using weighted linear regression and a weight of $(Pred\Delta D_{C,i,j})^{-1}$. For each installation and 5-yr growth period, the ΔD for each tree on the fertilized plots was predicted by the calibrated untreated-plot equation, and the ratio of the tree's actual ΔD divided by the calibrated predicted ΔD was calculated:

$$SFR_{\Delta D,i,j} = \Delta D_{F,i,j} / (k_{SFj} Pred\Delta D_{C,i,j})$$

This ratio estimates the direct effect of fertilization on ΔD.

The $SFR_{\Delta D,i,j}$ data were combined across all installations ($SFR_{\Delta D}$). Graphs of the ratio across PN_1, YF_1, SI_{SP}, D, BAL, and BA indicated that western hemlock exhibited no response to nitrogen fertilization. For Douglas-fir, $SFR_{\Delta D}$ increased with the amount of PN_1 first applied and decreased with both YF_1 and SI_{DF}; no effect of D, BAL, or BA could be detected. After examining a number of alternatives, we found that the following equation form best characterized the impact of a single fertilization on the ΔD of Douglas-fir:

$$SFR_{\Delta D} = 1.0 + a_{10}(PN_1/800)^{a_{11}} e^{a_{12}YF_1 + a_{13}[(SI-4.5)/100]^2} \qquad [7]$$

This equation was fit to a reduced set of the $SFR_{\Delta D}$ data (Table 12) by nonlinear regression. Removed from the final modeling data were the 235 observations that came from plots fertilized with >450 PN.

Graphing the residuals from Eq. [7] across ownerships indicated that the fertilization response data from Forestry Canada's Shawnigan Lake installations were being substantially underpredicted. To verify this, Eq. [7] was modified as follows to include indicator variables for the Forestry Canada installations:

$$SFR_{\Delta D} = 1.0 + (a_{10} + a_{10,FC} I_{FC})(PN_1 / 800)^{a_{11}} e^{a_{12} YF_1 + (a_{13} + a_{13,FC} I_{FC})[(SI - 4.5)/100]^2} \qquad [8]$$

where

I_{FC} = 1.0 if the data came from a Forestry Canada fertilization installation at Shawnigan Lake

= 0.0 otherwise

The parameters were estimated by nonlinear regression. Both the $a_{10,FC}$ and the $a_{13,FC}$ parameters were significantly different at $P = 0.01$, which verified the graphical evidence that the Forestry Canada data differed from the other data sets. A graph of the resulting residuals for Eq. [8] indicated homogeneous variance; therefore, weighting was deemed unnecessary. The species-specific parameter estimates and their SEs for Eq. [7] that resulted from fitting Eq. [8] to the data are found in Table 20. Parameters $a_{10,FC}$ and $a_{13,FC}$ are not reported in Table 20 because their use is limited to one Forestry Canada installation.

Equation for a Single Thinning Combined with a Single Fertilization

The effect of a single thinning combined with a single fertilization on ΔD of Douglas-fir was modeled as a multiplier on the equations for untreated plots. The untreated-plot equation was first calibrated to the control plot(s) on the installation in order to reduce variation caused by between-plot differences in the ΔD relationship (Hann and Hanus 2002). This calibration was done by regressing the control-plot data from each Douglas-fir plot on the predicted values from the equation for untreated plots with weighted linear regression:

$$\Delta D_{C,i,j} = k_{ST\&SF,j} \, Pred\Delta D_{C,i,j}$$

where

$\Delta D_{C,i,j}$ = measured ΔD for Douglas-fir trees with either measured or predicted CRs on all of the untreated plots in the j^{th} installation that included data from a single thinning combined with a single fertilization

Table 20. Parameters and asymptotic standard errors for predicting the response of 5-yr diameter-growth rate (ΔD) to fertilization in Douglas-fir and western hemlock, Eqs. [7] and [11].

Parameter/ Standard error	Douglas-fir	Western hemlock
a_{10}	0.850838693	0.0
SE(a_{10})	(0.10016556800)	(NA)
a_{11}	1.0	0.0
SE(a_{11})	(NA)	(NA)
a_{12}	-0.199222980	0.0
SE(a_{12})	(0.01540426706)	(NA)
a_{13}	-0.587552490	0.0
SE(a_{13})	(0.10665511768)	(NA)

NA: Not applicable.

$Pred\Delta D_{C,i,j}$ = predicted ΔD from Eq. [5] for the Douglas-fir trees with either measured or predicted *CRs* from all of the untreated plots on the j^{th} installation that included data from a single thinning combined with a single fertilization

$k_{ST\&SFj}$ = untreated Douglas-fir tree calibration for all of the untreated plots on the j^{th} installation that included data from a single thinning combined with a single fertilization

i = $1,\ldots,n_j$

n_j = the number of Douglas-fir sample trees with either measured or predicted CRs from all of the untreated plots on the j^{th} installation that included data from a single thinning combined with a single fertilization

The values of $k_{ST\&SFj}$ were estimated by using weighted linear regression and a weight of $(Pred\Delta D_{C,i,j})^{-1}$. For each installation and 5-yr growth period, the ΔD for each Douglas-fir tree on the thinned and fertilized plots was then predicted by the calibrated untreated-plot equation, and the ratio of the tree's actual ΔD to the calibrated predicted ΔD was calculated:

$$ST\&SFR_{\Delta D,i,j} = \Delta D_{T\&F,i,j} / (k_{ST\&SFj} Pred\Delta D_{C,i,j})$$

This ratio estimates the direct effect of a single thinning combined with a single fertilization on ΔD.

The $ST\&SFR_{\Delta D,i,j}$ data (Table 13) were then combined across all installations ($ST\&SFR_{\Delta D}$). We removed the Forestry Canada fertilization data at Shawnigan Lake because, as detailed above, it was significantly different from the other data. This reduced the number of observations available for modeling from 3,724 to 1,693. Most of the remaining data had predicted, rather than measured, CRs. Because of the impact of thinning on crown recession, we further restricted the data to those observations in which YT_1 and YF_1 at the start of the growth period were ≤1, resulting in a final modeling data set of just 270 observations from 18 installations.

We hypothesized that $ST\&SFR_{\Delta D}$ could be adequately predicted by the product of Eq. [6] (single thinning effect) times Eq. [7] (single fertilization effect). The following model form was used to evaluate this hypothesis:

$$ST\&SFR_{\Delta D} = [1.0 + a_8 (PREM_{\Delta D}) e^{a_9 YT_1}][1.0 + a_{10}(PN_1/800)^{a_{11}} e^{a_{12} YT_1 + a_{13}[(SI-4.5)/100]^2 + a_{14} X_8}] \quad [9]$$

where

$$X_8 = [(PREM_{\Delta D}) e^{a_9 YT_1}]^{1/2}$$

This formulation uses the parameters previously determined for Eq. [6] (Table 19) and Eq. [7] (Table 20) and allows the fertilization response to change with the form of the thinning, as reflected by the term $a_{14}X_8$ in Eq. [9]. If parameter a_{14} in Eq. [9] = 0, Eq. [9] reduces to the product of Eq. [6] and Eq. [7].

Equation [9] was fit to the reduced $ST\&SFR_{\Delta D}$ data set by nonlinear regression. The resulting value for a_{14} was 0.0349329987 (SE 0.17440617121), which was not significantly different from 0 at $P = 0.05$. Therefore, the effect of a single thinning combined with a single fertilization on ΔD of Douglas-fir was adequately characterized by the product of Eqs. [6] and [7].

Equation for Multiple Thinnings

The effect of multiple thinnings on ΔD of Douglas-fir and western hemlock was modeled as a multiplier on the equations for untreated plots. The untreated-plot equation was first calibrated to the control plot(s) found on each installation in order to reduce variation caused by between-plot differences in the ΔD relationship (Hann and Hanus 2002). This calibration was done by regressing each plot's control-plot data with a measured CR on the predicted values from Eq. [5] by weighted linear regression:

$$\Delta D_{C,i,j} = k_{MT,j} Pred\Delta D_{C,i,j}$$

where

$\Delta D_{C,i,j}$ = measured ΔD for trees with measured CR on all of the untreated plots in the j^{th} installation that included multiple-thinning data

$Pred\Delta D_{C,i,j}$ = predicted ΔD from Eq. [5] for the trees with measured CR from all of the untreated plots on the j^{th} installation that included multiple-thinning data

$k_{MT,j}$ = untreated tree calibration for all of the untreated plots on the j^{th} installation that included multiple-thinning data with measurements of CR

i = $1,\ldots,n_j$

n_j = the number of sample trees with measured CRs from all of the untreated plots on the j^{th} installation that included multiple-thinning data

The values of $k_{MT,j}$ were estimated by using weighted linear regression and a weight of $(Pred\Delta D_{C,i,j})^{-1}$. For each installation and 5-yr growth period, the ΔD for each tree on the multiply thinned plots was predicted by the calibrated untreated-plot equation, and the ratio of the tree's actual ΔD to the calibrated predicted ΔD was calculated:

$$MTR_{\Delta D,i,j} = \Delta D_{MT,i,j} / (k_{MT,j} Pred\Delta D_{C,i,j})$$

This ratio estimates any additional effect of multiple thinnings on ΔD not reflected in the values of the tree and plot attributes incorporated in the equation for untreated plots. The $MTR_{\Delta D,i,j}$ data were then combined across all installations ($MTR_{\Delta D}$).

Equation [6] predicts that the effect of a single thinning on ΔD exponentially declines as YT_1 increases. Therefore, one way to characterize multiple thinnings would be to "discount" the *BAR*s in more distant thinnings forward to the time of the most recent thinning and to add these discounted *BAR*s to BAR_1 and to $BABT_1$. Mathematically, the discounted *BAR* for the i^{th} thinning would be computed by

$$Discounted\ BAR_i = BAR_i e^{a_9(YT_i - YT_1)}$$

The effect of one or more thinnings would, therefore, be predicted by

$$TR_{\Delta D} = 1.0 + a_8[PREM_{\Delta D}]e^{a_9 YT_1} \qquad [10]$$

$$PREM_{\Delta D} = \frac{BAR_1 + \sum_{i=2}^{nt} BAR_i e^{a_9(YT_i - YT_1)}}{BABT + \sum_{i=2}^{nt} BAR_i e^{a_9(YT_i - YT_1)}}$$

where

BAR_i = BA removed in i^{th} thinning

$BABT$ = BA before most recent thinning

YT_i = number of years since i^{th} thinning

nt = number of thinnings

Equation [10] has been structured in a manner that reduces to the form of Eq. [6] when only one thinning is applied. We therefore fit Eq. [10] to the combined Douglas-fir $STR_{\Delta D}$ data from singly thinned plots (Table 11) and $MTR_{\Delta D}$ data from multiply thinned (Table 14) plots ($TR_{\Delta D}$), using nonlinear regression and the parameters from Eq. [6] as starting values. A graph of the resulting residuals indicated homogeneous variance; therefore, weighting was deemed unnecessary. Examination of the resulting fit indicated that the equation appeared to adequately characterize the effect of one or more thinnings for Douglas-fir.

The multiple-thinning data for western hemlock came from just two installations (Table 14) and were judged inadequate for fitting Eq. [10]. Therefore, the parameters from the fit to Eq. [6] were used in Eq. [10] to characterize the effect of one or more thinnings for western hemlock.

Table 21. Parameters and asymptotic standard errors for predicting the response of 5-yr diameter-growth rate (ΔD) to single and multiple thinnings in Douglas-fir and western hemlock, Eq. [10].

Parameter/ Standard error	Douglas-fir	Western hemlock
a_8	0.6203827985	0.723095045
SE(a_8)	(0.01984718404)	(NA)
a_9	-0.2644085320	-0.2644085320
SE(a_9)	(0.02318096359)	(NA)

NA: Not applicable.

The species-specific parameter estimates for Eq. [6] (Table 19) and Eq. [10] (Table 21) were different, and their SEs indicate that the differences are probably statistically significant. However, application of Eq. [10] and the parameters in Table 21 to the single-thinning data showed no noticeable trends when the residuals were plotted over YT_1 and $PREM_1$. Therefore, Eq. [10] and its parameters in Table 21 were judged adequate for characterizing the treatment response from both single and multiple thinnings.

Equation for Multiple Fertilizations

Equation [7] predicts that the effect of a single fertilization on ΔD of Douglas-fir declines exponentially as YF_1 increases. Therefore, one way to characterize multiple fertilizations would be to "discount" the *PN*s in more distant fertilizations forward to the time of the most recent fertilization and to add these discounted *PN*s to PN_1. Mathematically, the discounted *PN* for the i[th] fertilization would be computed by

$$Discounted\ PN_i = PN_i e^{a_{12}(YF_i - YF_1)}$$

The effect of one or more fertilizations ($FR_{\Delta D}$) would, therefore, be predicted by

$$FR_{\Delta D} = 1.0 + MFR_{\Delta D} \times \Delta DMOD_F \qquad [11]$$

$$MFR_{\Delta D} = a_{10}[(PN_1/800) + \sum_{i=2}^{nf}(PN_i/800)e^{a_{12}(YF_i - YF_1)}]^{a_{11}} e^{a_{13}[(SI_{DF}-4.5)/100]^2}$$

$$\Delta DMOD_F = e^{a_{12}YF_1}$$

Equation [11] has been structured in a manner that reduces to the form of Eq. [7] when only one fertilization is applied. Unfortunately, the Douglas-fir data sets (Table 15) available for multiple fertilizations were too small either to fit the parameters of Eq. [11] or to evaluate the use of the parameters from Eq. [7].

Equation for Multiple Thinnings Combined with Multiple Fertilizations

The effect of one or more thinnings combined with one or more fertilizations on ΔD of Douglas-fir was modeled as a multiplier on the untreated-plot equations. We assumed that our approach to estimating response on plots receiving a single thinning and a single fertilization could be applied to the multiple-thinning and multiple-fertilization situation. Therefore, the response of plots receiving multiple thinnings and multiple fertilizations was predicted by the product of Eq. [10] (multiple-thinning effect) times Eq. [11] (multiple-fertilization effect). Unfortunately, a lack of Douglas-fir data for multiply thinned and fertilized plots (Table 16) precluded the evaluation of this approach.

Discussion

The final, full equation for predicting ΔD is

$$\Delta D = \Delta D_C \bullet TR_{\Delta D} \bullet FR_{\Delta D}$$

ΔD_C is predicted by Eq. [5] and the parameters found in Table 17. $TR_{\Delta D}$ is predicted by Eq. [10] and the parameters found in Table 21. $FR_{\Delta D}$ is predicted by Eq. [11] and the parameters found in Table 20.

For untreated plots, this equation predicts that ΔD first increases and then decreases with an increase in D, that ΔD increases with an increase in CR and SI_{SP}, and that ΔD decreases with an increase in BAL and BA. These results are in agreement with those of Hann and Larsen (1991) and Zumrawi and Hann (1993).

The geographic area where the data used to develop the Zumrawi and Hann (1993) equation for Douglas-fir come from falls within the geographic area for the SMC study. Predictions from the Zumrawi and Hann (1993) Douglas-fir equation were very similar to predictions from Eq. [5] for Douglas-fir.

The thinning-effects modifier expressed in Eq. [10] and the associated parameters for each species in Table 21 predict that ΔD increases with $PREM_{\Delta D}$ (the proportion of the BA removed) and decreases with YT with most of the thinning effect gone once YT reaches 10 yr (Figure 1). Wang (1990) found that the ΔD equation for Douglas-fir in the southwest Oregon version of ORGANON, developed data from basically untreated plots, also underestimated the ΔD of trees on thinned plots for the first 5-yr growth period since thinning. He further reported that the amount of underestimation increased with the amount of BA removed in thinning, supporting the findings of this study.

Thinning reduces BA of the residual stand and, depending on the type of thinning (i.e., from above, from below, or proportional), can reduce BAL of the residual trees. These reductions will cause an increase in predicted ΔD from the equation developed with untreated-plot data. Our results indicate, however, that the actual increase in ΔD is greater than can be explained by these factors alone. The additional increase may be due to one or more of the following:

(1) Hann and Hanus (2002) have found that trees with various types of damage exhibit reduced ΔD when compared with undamaged trees. Thinning usually targets the removal of damaged trees first and, therefore, changes the damage composition of the residual stand, which should lead to increased ΔD.

(2) The HCB Eq. [2] shows inhibited crown recession immediately following thin-

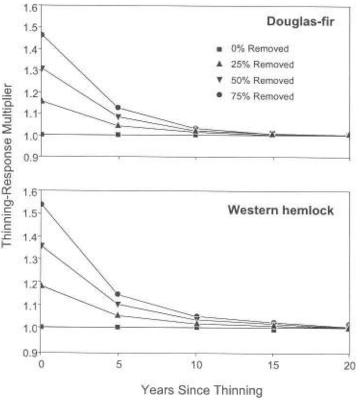

Figure 1. Thinning-response multipliers for 5-yr diameter-growth rate (ΔD) in Douglas-fir and Western hemlock.

ning, and, as a result, *CL* is longer than would be expected in an untreated stand with the same *BA* and *CCFL*. Therefore, average *CL* over the 5-yr growth period will be longer than expected in an untreated stand. The larger crowns should produce a greater ΔD.

(3) The removal of trees can reduce competition for moisture and nutrients. Because the fine-root systems of trees are highly dynamic (Santantonio 1982; Waring and Schlesinger 1985) and root graft between both cut and uncut trees is prevalent (Eis 1972), the residual trees can very quickly take advantage of the soil space made available by thinning. The resulting improvement in the availability of moisture and nutrients can cause the stomata of the crown to remain open longer during the day, increasing photosynthesis. Thus, trees can demonstrate an increased growth rate after thinning even before crown size increases.

We found no response of ΔD to fertilization in western hemlock. Olson et al. (1979) reported that western hemlock response to nitrogen fertilization was extremely variable between locations and that the average response was low. More recently, Stegemoeller et al. (1990, p. 10) concluded that "western hemlock stands have not shown any consistent evidence of response to fertilization, whether thinned or unthinned".

The fertilization-effects modifier in Eq. [11] and the associated parameters for Douglas-fir (Table 20) predict that ΔD increases with *PN* and decreases with *YF*, with most of the fertilization effect gone once *YF* reaches 15 yr. The size of the increase depends on the SI_{DF} of the plot, with plots of lower site quality showing greater increases (Figure 2). Numerous previous studies have reported an increase in ΔD or basal area growth rate of Douglas-fir trees (e.g., Shafii et al. 1990; Wang 1990; Carter et al. 1998; Shen et al. 2000) or an increase in *BA* growth or volume growth of Douglas-fir stands (e.g., Curtis et al. 1981; Miller et al. 1988; Zhang and Moore 1993) following fertilization. Five of these studies reported that the response increased with *PN* (Curtis et al. 1981; Miller et al. 1988; Wang 1990; Zhang and Moore 1993; Shen et al. 2000), though the increase between $PN = 200$ and $PN = 400$ was not statistically different in the studies of Wang (1990) and Zhang and Moore (1993). We found that fertilization response increased at a constant rate with *PN*. The work of Curtis et al. (1981) and Miller et al. (1988) predicted that fertilization response for stand-level attributes increased at a decreasing rate with *PN*.

Curtis et al. (1981) and Miller et al. (1988) reported that the annual response to fertilization increased, peaked, and then decreased with *YF*, whereas Wang (1990) showed

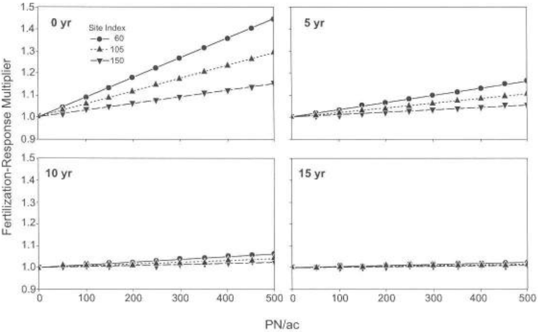

Figure 2. Douglas-fir fertilization-response multiplier for 5-yr diameter-growth rate (ΔD) 0, 5, 10, and 15 yr after fertilization.

that 5-yr response to fertilization decreased exponentially with an increase in *YF*. The response peaked between 2 and 3.5 yr in the Curtis et al. (1981) study and between 3 and 5 yr, with a later peak on lower values of SI_{DF}, in the Miller et al. (1988) study. It is likely, therefore, that these two studies would also show an exponential decrease with an increase in *YF* if summarized by 5-yr, rather than annual, periods.

Curtis et al. (1981) and Zhang and Moore (1993) reported that fertilization response decreased with an increase in SI_{DF}, as we found in this study. Miller et al. (1988) found that fertilization response increased, peaked, and then decreased with SI_{DF}, with the location of the peak varying by stand age.

We could detect no variation in fertilization response associated with plot density, tree size, or tree position within the plot. In contrast, other studies have reported that the fertilization response varied by initial relative density (Miller et al. 1988), by *N* (Zhang and Moore 1993), by tree size (Shafii et al. 1990), and by tree position within the plot (Shafii et al. 1990; Shen et al. 2000). However, Shafii et al. (1990) did not statistically test whether the tree size and position effects were significantly different from those usually found in untreated-plot data, and Shen et al. (2000) found tree position to be statistically significant from the untreated-plot effect in only 1 of 10 subsets of the data ($P = 0.01$).

Finally, we found that the combined effect of applying both thinning and fertilization on ΔD could be adequately characterized by the product of the thinning modifier (Eq. [10] and the parameters in Table 21) times the fertilization modifier (Eq. [11] and the parameters in Table 20). As a result, the percent increase due to a combined treatment is greater than the sum of the percent increases for each treatment alone. For example, a predicted 20% increase due to fertilization alone combined with a predicted 10% increase due to thinning alone would result in a 32% increase.

In coastal Douglas-fir, Miller et al. (1986) reported that increases in live-stand basal area were greater after fertilization in combination with thinning than after fertilization. Curtis et al. (1981) also used a multiplicative approach, such as ours, to characterize this effect for gross volume, gross BA, and net QMD growth rates of Douglas-fir stands.

Five-Year Height-Growth Rate

Data Description

All 5-yr plot-level dominant height-growth-rate data ($\Delta H40$) and all 5-yr tree-level ΔH data for Douglas-fir and western hemlock were extracted from the data base. The resulting data were divided into six groups for each species:

"dominant untreated" plot data, consisting of all $\Delta H40$ measurements from plots that had been untreated. The resulting data sets, including variables used in the final $\Delta H40$ equations, are described in Table 22.

Table 22. Description of the top-height growth-rate ($\Delta H40$) data sets for Douglas-fir and western hemlock on untreated plots. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual plot	*NOb = 1,016*		*NOb = 91*	
$\Delta H40$	9.48	0.8–21.6	10.35	3.3–17.5
BH AGE	34.2	8.0–81.0	30.7	6.2–70.0
Installation	*NOb = 196*		*NOb = 6*	
SI_{SP}	108.9	56.1–156.0	111.6	91.9–123.6

"dominant single fertilization" plot data, consisting of all $\Delta H40$ measurements from plots that had been fertilized only once. The resulting data sets, including variables used in the final $\Delta H40$ equations, are described in Table 23.

Table 23. Description of the top-height growth-rate ($\Delta H40$) data set for Douglas-fir and western hemlock plots receiving a single fertilization. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual plot	*NOb = 1,010*		*NOb = 164*	
$\Delta H40$	10.23	0.5–20.4	9.48	2.3–15.2
BH AGE	34.9	10.6–85.1	35.2	11.6–70.2
nf	1.0	1.0–1.0	1.0	1.0–1.0
YF_1	2.5	0.0–12.0	2.2	0.0–6.0
PN_1	279.3	33.0–803.0	353.0	200.0–803.0
Installation	*NOb = 163*		*NOb = 32*	
SI_{SP}	108.8	56.1–156.0	111.2	92.8–127.0

"dominant multiple fertilization" plot data, consisting of all $\Delta H40$ measurements from plots that had been fertilized more than once. The resulting data sets, including variables used in the final $\Delta H40$ equations, are described in Table 24.

Table 24. Description of the top-height growth-rate ($\Delta H40$) data sets for Douglas-fir and western hemlock plots receiving more than one fertilization. The means were computed from the number of observations reported for each variable.

	Douglas-fir			Western hemlock	
Variable	Mean	Range		Mean	Range
Individual plot	*NOb = 3*			*NOb = 2*	
$\Delta H40$	6.53	5.1–7.5		9.80	8.3–11.3
BH AGE	47.0	47.0–47.0		27.9	27.9–27.9
nf	2.7	2.0–3.0		3.0	3.0–3.0
YF_1	0.0	0.0–0.0		0.0	0.0–0.0
PN_1	200.0	200.0–200.0		300.0	200.0–400.0
Installation	*NOb = 1*			*NOb = 1*	
SI_{SP}	116.0	116.0–116.0	SI_{WH}	100.0	100.0–100.0

"untreated" tree data, consisting of all ΔH tree measurements with actual CR measurements at the start of the growth period from the untreated control plots. The resulting data sets, including variables used in the final ΔH equations, are described in Table 25.

Table 25. Description of the 5-yr height-growth-rate (ΔH) data sets for Douglas-fir and western hemlock trees with measured crown ratios on untreated plots. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 3,200*		*NOb = 873*	
ΔH	9.61	0.2–30.7	13.09	0.2–20.3
H	52.31	7.0–147.9	24.40	5.2–116.5
CR	0.63	0.09–0.97	0.72	0.13–0.99
CCH	23.3	0.0–364.4	24.4	0.0–307.6
Individual plot	*NOb = 139*		*NOb = 29*	
$\Delta H40$	10.57	1.5–19.0	13.01	7.2–23.0
BH AGE	26.6	11.0–60.5	27.6	6.2–44.1
Installation	*NOb = 24*		*NOb = 7*	
SI_{SP}	115.0	77.6–142.0	109.9	91.9–123.6

"single thinning" tree data, consisting of all ΔH tree measurements with actual *CR* measurements at the start of the growth period from plots that had been thinned only once. The resulting data sets for Douglas-fir, including variables used in the final ΔH equations, are described in Table 26 for Douglas-fir. No data from western hemlock met these criteria.

Table 26. Description of the 5-yr height-growth-rate (ΔH) data set for Douglas-fir trees with measured crown ratios receiving a single or more than one thinning. The means were computed from the number of observations reported for each variable.

	1 Thinning		>1 Thinning	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 2,113*		*NOb = 4,495*	
ΔH	7.33	0.8–30.0	10.20	0.1–28.2
H	46.05	9.0–178.0	81.25	18.0–167.0
CR	0.65	0.14–0.91	0.55	0.08–0.90
CCH	10.98	0.0–250.2	8.99	0.0–190.1
Individual plot	*NOb = 137*		*NOb = 412*	
$\Delta H40$	9.00	1.7–18.9	11.28	3.9–19.5
BH AGE	26.9	11.0–65.0	33.3	16.0–65.0
nt	1.0	1.0–1.0	4.6	2.0–6.0
YT_1	8.0	0.0–30.1	3.3	0.0–20.0
$PREM_1$	0.503	0.043–0.922	0.140	0.002–0.463
Installation	*NOb = 14*		*NOb = 11*	
SI_{SP}	108.6	77.6–142.0	123.5	85.8–137.9

"multiple thinning" tree data, consisting of all ΔH tree measurements with actual *CR* measurements at the start of the growth period from plots that had been thinned more than once. The resulting data sets for Douglas-fir, including variables used in the final ΔH equations, are also described in Table 26. No data from western hemlock met these criteria.

Data Analysis and Results

The "potential/modifier" approach of Hann and Ritchie (1988) was taken to model ΔH. The potential ΔH ($P\Delta H$) of the tree is first predicted and then a multiplicative modifier is used to adjust $P\Delta H$ to the vigor and competitive status of the tree:

$$\Delta H = (P\Delta H)(\Delta HMOD)$$

where

$\Delta HMOD$ = height-growth-rate modifier function and

$P\Delta H$ is "...a theoretical estimate of the growth rate of a dominant tree of that size..." (Wensel et al. 1987).

Equations for Potential Height Growth of Untreated Plots

The dominant height-growth ($H40$) equations of King (1966) and Bruce (1981) for Douglas-fir were evaluated against the measurements of actual $H40$ from the control plots to determine which to use for estimating $P\Delta H$ for untreated trees. The control plots from both the Levels-of-Growing-Stock (LOGS) installations (Williamson and Staebler 1971) and the Type II SMC installations (Chappell and Osawa 1991) were chosen for this evaluation because their total area was at least 0.5 ac. In addition, the LOGS plots had measurement periods at least 20 yr long. Based on this evaluation, Bruce's (1981) equation was chosen as better representing the $H40$ of Douglas-fir.

For western hemlock, the $H40$ equation of Bonner et al. (1995) was chosen over that of Wiley (1978). The Bonner et al. (1995) equation was developed using most of the western hemlock data available for this analysis. As a result, any comparison would most likely demonstrate its superiority over that of Wiley (1978).

$P\Delta H$ for untreated trees was then calculated from these equations in the following manner:

$$P\Delta H_C = f_{SP}[SI_{SP},(GEA+5.0)]-H \qquad [12]$$

where

$P\Delta H_C$ = potential height-growth rate of untreated trees

f_{SP} = the $H40$ function for species "SP"

GEA = the calculated growth-effective age for the tree

GEA is the age of a dominant tree (as defined by membership in the 40 largest-diameter trees per ac) with the same height and on the same site as the tree of interest (Hann and Ritchie 1988):

$$GEA = f_{SP}^{-1}[SI_{SP},H]$$

Equation for Potential Height Growth of Trees with a Single Fertilization

The data available for modeling the effect of fertilization on ΔH were limited on most installations because (1) many plots were small (i.e., ~0.1 ac), (2) heights were measured only on the 40 largest-diameter trees per ac (i.e., the dominant trees), and (3) no measurements of *CR* were taken. Because of these limitations, it was necessary to model the effect of fertilization as a multiplier on the 5-yr growth in *H40* for the control plots:

$$P\Delta H_F = (\Delta H40_C)(FR_{\Delta H40}) \quad [13]$$

$$\Delta H40_C = f_{SP}[SI_{SP}, (PGEA + 5.0)] - H40$$

where

$P\Delta H_F$ = potential height-growth rate of fertilized trees

$FR_{\Delta H40}$ = fertilization response

$\Delta H40_C$ = height-growth rate of *H40* for untreated plots

PGEA = the calculated growth effective age for the plot

= $f^{-1}_{SP}[SI_{SP}, H40]$

$\Delta H40_c$ should be an unbiased estimator of $P\Delta H_c$ for the dominant trees in the stand.

We first evaluated whether predicted $\Delta H40_C$ ($Pred\Delta H40_C$) from Eq. [13] agreed with the measured $\Delta H40_C$ on the control plots (Table 22). Of concern was the potential impact of small plot size used in most of the fertilized installations on the estimates of *H40* (Garcia 1998, Magnusson 1999) and, therefore, of *SI*, and whether the *H40* equations of Bruce (1981) and Bonner et al. (1995) fully characterized $\Delta H40_C$ over the full range of *SI* found in the fertilization data. (The previous analysis of *H40* had merely established which of two alternatives was better for each of the two species.)

This analysis was done by forming the ratio of measured $\Delta H40_C$ to $Pred\Delta H40_C$ and plotting this ratio across SI_{SP} of the installation and breast-height age of the plot. No trends were observed for western hemlock. A trend across SI_{DF} was observed for Douglas-fir, but there were no trends for data from installations in which the total area in untreated plots was at least 0.5 ac. This finding agreed with the previous analysis that resulted in our selecting Bruce's (1981) *H40* equation for Douglas-fir. Data from installations with <0.5 ac in control plots were primarily from the fertilization studies.

An equation was then formed to correct the trend in the Douglas-fir data from the fertilization installations. Preliminary fits to the data indicated that the trend of the data from the British Columbia Ministry of Forestry was somewhat different from that of the other fertilization data sets. This led to the following correction equation:

$$C\Delta H40_C = (\Delta H40_C)(1.0 + 0.358324716e^{-[0.009237676(SI_{DF}) - 0.002104279(I_{BCMF})(SI_{DF})]^{10}})$$

where

$C\Delta H40_C$ = corrected $\Delta H40_C$

I_{BCMF} = 1.0 if data came from the British Columbia Ministry of Forestry fertilization installation

= 0.0 otherwise

To evaluate whether there was a single fertilization effect on $\Delta H40$, the ratio $SFR_{\Delta H40}$ was formed by dividing measured $\Delta H40_F$ (Table 23) by predicted $C\Delta H40_C$ (for western hemlock, $C\Delta H40_C = \Delta H40_C$), using the data from all plots that received a single fertilization. The $SFR_{\Delta H40}$ data were then combined across all installations. Graphs of the ratio across PN_I, YF_I, and SI_{SP} indicated that western hemlock exhibited no response to nitrogen fertilization. For Douglas-fir, $SFR_{\Delta H40}$ increased with the amount of PN_I first applied and decreased with both YF_I and SI. After examining many alternatives, we found that the following equation form best characterized the impact of a single fertilization on the $\Delta H40$ of Douglas-fir:

$$SFR_{\Delta H40} = 1.0 + (PN_1/800)^{1/3} e^{b_1 YF_1 + b_2[(SI_{DF} - 4.5)/100]^{1.5}} \quad [14]$$

Table 27. Parameters and asymptotic standard errors for predicting the potential 5-yr height-growth rate (ΔH) of fertilized Douglas-fir and western hemlock, Eqs. [14] and [15].

Parameter/ Standard error	Douglas-fir	Western hemlock
b_1	-1.0	0.0
SE(b_1)	(NA)	(NA)
b_2	-2.328442529	0.0
SE(b_2)	(0.174139441)	(NA)

NA: Not applicable

This equation was fit to the $SFR_{\Delta H40}$ data with nonlinear regression. A graph of the resulting residuals indicated homogeneous variance, so weighting was deemed unnecessary. The b_1 parameter could not be estimated by nonlinear regression, indicating that the impact of a single fertilization was over after 5 yr. Therefore, b_1 was set to -1.0, which results in a value of 1.0 for $SFR_{\Delta H40}$ when $YF_I \geq 5$. The parameter estimates and their SEs for Eq. [14] are found in Table 27.

Equation for Potential Height Growth of Trees with Multiple Fertilizations

Equation [14] predicts that the effect of a single fertilization on $\Delta H40$ declines exponentially as YF_I increases. Therefore, one way to characterize multiple fertilizations would be to "discount" the PNs in more distant fertilizations to the time of the most recent fertilization and to add them to PN_I. Mathematically, the discounted PN for the i^{th} fertilization would be computed by

$$Discounted\ PN_i = PN_i e^{3.0 b_1 (YF_i - YF_1)}$$

The effect of one or more fertilizations ($FR_{\Delta H40}$) would, therefore, be predicted by

$$FR_{\Delta H40} = 1.0 + [(PN_1/800) + \sum_{i=2}^{nf} (PN_i/800) e^{3.0 b_1 (YF_i - YF_1)}]^{1/3} [e^{b_1 YF_1 + b_2[(SI_{DF} - 4.5)/100]^{3/2}}] \quad [15]$$

Equation [15] reduces to the form of Eq. [14] when only one fertilization is applied. Unfortunately, the data set available for multiple fertilizations (Table 24) was too small either to fit the parameters of Eq. [15] or to evaluate the use of the parameters from

Eq. [14]. Therefore, we assumed that parameter estimates and their SEs for Eq. [15] are the same as for Eq. [14] (Table 27).

Effect of Thinning on *H40*

Thinning can affect the value of *H40*, particularly if the thinning was conducted from above in a manner that removed dominant trees. To evaluate whether thinning affected top height, the *H40* immediately after thinning was subtracted from the *H40* immediately before thinning for those plots receiving one or more thinnings. The average of this difference was 0.12 ft for Douglas-fir and 0.03 ft for western hemlock. Both values were within the measurement precision of *H40*; therefore, we concluded that the thinnings applied in the data set available for this study did not impact *H40*.

Modifier Equation for Height Growth of Untreated Trees

The following equation form was fit to the "untreated" tree data with *CR* (Table 25) for both species by nonlinear regression:

$$\Delta HMOD_C = b_3[b_4 e^{b_5 CCH} + (e^{b_6 CCH^{1/2}} - b_4 e^{b_5 CCH}) e^{-b_7 (1.0-CR)^2 e^{b_8 CCH^{1/2}}}] \quad [16]$$

where

$\Delta HMOD_C$ = ΔH modifier for trees on untreated plots

= $\Delta H_C / PredP\Delta H_C$

$PredP\Delta H_C$ = predicted potential ΔH for trees on untreated plots using Eq. [12]

This equation form has been previously used to model ΔH of predominantly untreated stands in SWO-ORGANON (Hann and Ritchie 1988; Ritchie and Hann 1990). As in the previous work, the residuals about the equation appeared to be homogeneous.

Examination of the resulting parameters indicated that the values for Douglas-fir were all significantly different from zero ($P = 0.05$) and that the parameters had the correct signs and were of reasonable magnitude. The western hemlock parameters, on the other hand, exhibited many problems. After examination of numerous alternatives, the final set of parameters was determined by fixing four of the six parameters to values judged to be reasonable in sign and magnitude and estimating the remaining two by nonlinear regression. These problems probably were caused by the small data set available with measured *CR*. The parameter estimates and their SEs for Eq. [16] are found in Table 28.

Table 28. Parameters and asymptotic standard errors for predicting the vigor-and-competition modifier to potential 5-yr height-growth rate (ΔH) of untreated Douglas-fir and western hemlock, Eq. [16].

Parameter/ Standard error	Douglas-fir	Western hemlock
b_3	1.052301385	1.03
SE(b_3)	(0.01272028596)	(NA)
b_4	0.638569239	1.0
SE(b_4)	(0.03774281231)	(NA)
b_5	-0.005328221	-0.0056949357
SE(b_5)	(0.00064201525)	(0.00066717738)
b_6	-0.049351159	-0.0018047267
SE(b_6)	(0.00689948052)	(0.0045432342)
b_7	0.464049843	6.1978
SE(b_7)	(0.14670785979)	(NA)
b_8	0.485384235	0.0
SE(b_8)	(0.06903822266)	(NA)

NA: Not applicable.

Modifier Equation for Height Growth of Trees After a Single Thinning

Individual tree measurements of ΔH and CR from plots with a single thinning were available only for Douglas-fir. Therefore, the effects of a single thinning on Douglas-fir ΔH was modeled as a multiplier on the individual-tree modifier equation:

$$\Delta HMOD_T = (\Delta HMOD_C)(STR_{\Delta H})$$

where

$\Delta HMOD_T$ = modifier function for trees receiving a single thinning

$STR_{\Delta H}$ = response for a single thinning

To evaluate whether single thinning affected ΔH, the ratio $STR_{\Delta H,i,j}$ was formed by dividing measured $\Delta H_{T,i,j}$ (Table 26) by the product $(PredP\Delta H_{C,i,j})$ $(\Delta HMOD_C)$, using the data from the i^{th} tree on plots from the j^{th} installation that included a single thinning. The $STR_{\Delta H,i,j}$ data were then combined across all installations ($STR_{\Delta H}$). Graphs of the ratio across $PREM_I$, YT_I, SI, CCH, CR, and H indicated that a single thinning reduced ΔH and that the reduction increased with the amount of $PREM_I$ removed and decreased with YT_I. After examining many alternatives, we found that the following equation form best characterized the impact of a single thinning on ΔH of Douglas-fir:

$$STR_{\Delta H} = 1.0 + b_9[PREM_{\Delta H}]^{b_{10}}[e^{b_{11}YT_1}] \quad [17]$$

where

$$PREM_{\Delta H} = \frac{BAR_1}{BABT}$$

An attempt was made to fit this equation to the full $STR_{\Delta H}$ data (Table 26) by nonlinear regression. Unfortunately, the program would not converge to an estimate of the final parameters. Further examination indicated substantial variation in the data, particularly those with $YT_I > 15$ yr. Convergence was achieved when the data with $YT_I > 15$ were removed from the data set. A graph of the resulting residuals indicated homogeneous variance; therefore, weighting was deemed unnecessary. The parameter estimates and their SEs for Eq. [17] are found in Table 29.

Table 29. Parameters and asymptotic standard errors for predicting the thinning-response changes to the vigor-and-competition modifier of potential 5-yr height-growth rate (ΔH) for Douglas-fir and western hemlock, Eqs. [17] and [18].

Parameter/ Standard error	Douglas-fir	Western hemlock
b_9	-0.3197415492	0.0
SE(b_9)	(0.00001422247)	(NA)
b_{10}	0.7528887377	1.0
SE(b_{10})	(0.07271920898)	(NA)
b_{11}	-0.2268800162	0.0
SE(b_{11})	(0.07878170031)	(NA)

NA: Not applicable.

Equation for a Single Thinning Combined with a Single Fertilization

The effect of a single thinning combined with a single fertilization on ΔH of Douglas-fir was modeled as a multiplier on the untreated equations. Given the lack of modeling data, we hypothesized that the multiplier could be adequately portrayed as the product of Eq. [14] (single fertilization effect) and Eq. [17] (single thinning effect).

Modifier Equation for Height Growth of Trees After Multiple Thinnings

The effect of multiple thinnings on ΔH of Douglas-fir was modeled as a multiplier on untreated ΔH. Data for western hemlock were insufficient. For each installation and 5-yr growth period, the ratio $MTR_{\Delta H,i,j}$ was formed by dividing the measured $\Delta H_{MT,i,j}$ for each tree on the multiply thinned plots with CR (Table 26) by the product ($PredP\Delta H_{C,i,j}$) (predicted $\Delta HMOD_C$). This ratio estimates the direct effect of multiple thinnings on ΔH. The $MTR_{\Delta H,i,j}$ data were then combined across all installations ($MTR_{\Delta H}$).

Equation [17] predicts that the effect of a single thinning on ΔH declines exponentially as YT_1 increases. Therefore, one way to characterize multiple thinnings would be to discount the BARs in more distant thinnings to the time of the most recent thinning and to add these discounted BARs to both BAR_1 and $BABT_1$ in order to form a discounted $PREM$. Mathematically, the discounted BAR for the i^{th} thinning would be computed by

$$Discounted\ BAR_i = BAR_i e^{\frac{b_{11}}{b_{10}}(YT_i - YT_1)}$$

The effect of one or more thinnings would, therefore, be predicted by

$$TR_{\Delta H} = 1.0 + b_9[PREM_{\Delta H}]^{b_{10}}[e^{b_{11}YT_1}] \qquad [18]$$

$$PREM_{\Delta H} = \frac{BAR_1 + \sum_{i=2}^{nt} BAR_i e^{\frac{b_{11}}{b_{10}}(YT_i - YT_1)}}{BABT + \sum_{i=2}^{nt} BAR_i e^{\frac{b_{11}}{b_{10}}(YT_i - YT_1)}}$$

Equation [18] reduces to the form of Eq. [17] when only one thinning is applied.

The $STR_{\Delta H}$ data for Douglas-fir from singly thinned plots and the $MTR_{\Delta H}$ data for Douglas-fir from multiply thinned plots (Table 26) were combined ($TR_{\Delta H}$). An attempt to fit Eq. [18] to the combined data set with nonlinear regression failed because the parameter estimates would not converge. Attempts to obtain convergence by reducing the combined data set (which was successful for the single thinning equation) also failed. The $MTR_{\Delta H}$ data for Douglas-fir was then divided by predictions from Eq. [18] (with the parameter estimates from Eq. [17]), and the resulting ratios were examined for trends. Eq. [18] (with the parameters from Eq. [17]) appeared to characterize the effect of multiple thinnings on ΔH of Douglas-fir adequately. The parameter estimates and their SEs for Eqs. [17] and [18] are found in Table 29.

Equation for Multiple Thinnings Combined with Multiple Fertilizations

The effect of one or more thinnings combined with one or more fertilizations on ΔH of Douglas-fir was modeled as a multiplier on the untreated-plot equations. Because of the lack of adequate modeling data, we hypothesized that the multiplier could be adequately predicted by the product of Eq. [15] (multiple fertilization effect) and Eq. [18] (multiple thinning effect).

Discussion

The final, full equation for predicting ΔH is

$$\Delta H = [(P\Delta H_C)(FR_{\Delta H40})] \bullet [(\Delta HMOD_C)(TR_{\Delta H})]$$

$P\Delta H_C$ is predicted by Eq. [12]. $FR_{\Delta H40}$ is predicted by Eq. [15] and the parameters in Table 28. $\Delta HMOD_C$ is predicted by Eq. [16] and the parameters in Table 28. $TR_{\Delta H}$ is predicted by Eq. [18] and the parameters in Table 29.

For untreated plots, this equation predicts that ΔH will increase, peak, and then decrease with *GEA*; increase with an increase in *SI*; decrease with an increase in *CCH* (a measure of vertical position and one-sided light competition); and increase with an increase in *CR*. These findings closely agree with the previous work of Hann and Ritchie (1988) and Ritchie and Hann (1990), who used the same model form for characterizing the ΔH for Douglas-fir in southwest Oregon. They also agree with the work of Ritchie and Hann (1986) and Wensel et al. (1987), both of whom used different model forms from that used in this study to characterize the ΔH for Douglas-fir in northwest Oregon and northern California, respectively.

However, the unweighted MSE for the fit of Eq. [16] in this study was more than double that reported by Hann and Ritchie (1988) and Ritchie and Hann (1990) for Douglas-fir. The most likely cause for this difference is the large measurement error that can occur when heights are measured on standing trees (e.g., Larsen et al. 1987, Williams et al. 1994), rather than felled trees, as was done by Hann and Ritchie (1988) and Ritchie and Hann 1990).

Because crown size is affected strongly by stand density (Curtis and Reukema 1970, Oliver and Larson 1996), it can be viewed as a surrogate for density. The results of this study indicate that if *CR* of Douglas-fir drops below 37%, even the tallest tree in the stand will not grow at the potential for the *SI*. This was not true for the very tolerant western hemlock (Hardin et al. 2001). The negative impact of density on $P\Delta H_C$ has been previously reported by Curtis and Reukema (1970) for Douglas-fir, Lynch (1958) for ponderosa pine, Alexander (1966) for lodgepole pine, and MacFarlane et al. (2000) for loblolly pine. All of these species are intermediate in tolerance or intolerant to competition (Hardin et al. 2001).

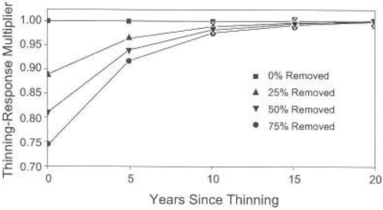

Figure 3. Douglas-fir thinning-response multiplier for 5-yr height-growth rate (ΔH).

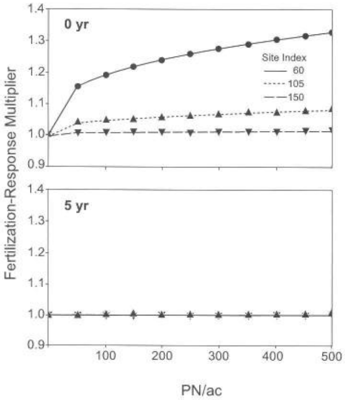

Figure 4. Douglas-fir fertilization-response multiplier for 5-yr height-growth rate (ΔH) at 0 and 5 yr after fertilization.

For Douglas-fir, Eq. [18] for $TR_{\Delta H}$ and the associated parameters in Table 29 predict a reduction in ΔH immediately following thinning, with the magnitude of the reduction increasing with the intensity of thinning. Most of the reduction is gone by about 10 yr after thinning (Figure 3). No reduction in ΔH due to thinning was found for western hemlock. The difficulties in fitting Eqs. [17] and [18] for both species may also have resulted from measurement error described earlier.

Reduced ΔH for Douglas-fir after thinning has been previously reported by Staebler (1956), Harrington and Reukema (1983), Maguire (1983), and DeBell et al. (2002). Harrington and Reukema (1983) found that the reduction lasted for 10 yr.

We found no fertilization $P\Delta H$ response for western hemlock. As with ΔD, this result is supported by the studies of Olson et al. (1979) and Stegemoeller et al. (1990).

The fertilization effects modifiers in Eqs. [14] and [15] and the associated parameters for Douglas-fir (Table 27) predict that 5-yr $P\Delta H$, as defined by the dominant trees on the installation, increases with PN and decreases with YF, with all of the fertilization effect gone after 5 yr. The size of the increase depends on the plot SI_{DP}, with plots of lower site quality showing greater increases (Figure 4). Several studies have reported an increase in $P\Delta H$ following fertilization (e.g., Curtis et al. 1981; Wang 1990). Curtis et al. (1981) reported that the response increased with PN, while Wang (1990) could not detect a statistically significant difference between fertilizing with $PN = 200$ and $PN = 400$.

Curtis et al. (1981) reported that the annual response to fertilization increased, peaked, and then decreased with YF, while Wang (1990) showed that 5-yr response to fertilization decreased exponentially with an increase in YF. The peak in the Curtis et al. (1981) study occurred between 2 and 3.5 yr, with a later peak occurring at lower values of SI_{DP}. The Curtis et al. (1981) study, therefore, also would likely show an exponential decrease with an increase in YF if summarized by 5-yr, rather than annual, periods.

Curtis et al. (1981) also reported that fertilization response of $P\Delta H$ decreased with an increase in SI_{DP}, as we found in this study. We found no trend in fertilization response of

$P\Delta H$ by plot density. Because of the structure of the modeling data available, we could not examine whether tree size or tree position within the plot affected fertilizer response.

As with ΔD, we found that the combined effect on ΔH of applying both thinning and fertilization could be adequately characterized by the product of the thinning modifier (Eq. [18] and the parameters in Table 29) and the fertilization modifier (Eq. [15] and the parameters in Table 27). As a result, the percent increase in ΔH resulting from a combined treatment is greater than the sum of the percent increases for each treatment alone.

The need to correct the $\Delta H40$ predictions from Bruce's (1981) equation was caused by the small acreage in control plots in the fertilization data sets. The true *SI* of the fertilization installations was often underestimated because the small number and size of the control plots decreased the likelihood that the trees with the largest 40 diameters at the site would be adequately sampled. Under this situation, the frequency of underestimation would increase as the size of the estimate of *SI* of the control plot(s) decreased (i.e., a *SI* estimate of 70 ft would more likely underestimate the true *SI* of the installation than would a *SI* estimate of 150 ft). This behavior is exactly what was found in this study.

Five-Year Mortality Rate

Data Description

All Douglas-fir and western hemlock PM data for trees with a growth period between 3 and 7 yr were extracted from the data base and divided into six groups for each species:

"untreated" data, consisting of all measurements from untreated control plots. *CR* was predicted by *HCB* Eq. [1] for western hemlock and by the *HCB* equation of Zumrawi and Hann (1989) for Douglas-fir. The resulting data sets, including variables used in the final mortality rate equations, are described in Table 30.

Table 30. Description of the 5-yr probability-of-mortality (*PM*) data set for Douglas-fir and western hemlock trees on untreated plots. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 153,660*		*NOb = 44,354*	
PM	0.0704	NA	0.0876	NA
D	7.07	0.1–67.1	4.86	0.1–25.8
PCR	0.48	0.13–0.97	0.53	0.16–1.00
BAL	115.1	0.0–400.2	162.1	0.0–416.9
Individual plot	*NOb = 1,766*		*NOb = 991*	
PLEN	5.18	3.0–7.0	5.18	3.0–7.0
BH AGE	35.4	8.0–87.0	33.6	6.2–85.1
BA	191.8	9.1–417.2	199.4	9.1–417.2
Installation	*NOb = 250*		*NOb = 195*	
SI_{SP}	111.2	56.1–156.0	103.4	43.0–138.1

"single thinning" data, consisting of all measurements from plots that had been thinned only once. *CR* was predicted for all trees by Eq. [2]. The resulting data sets, including variables used in the final mortality rate equations, are described in Table 31.

Table 31. Description of the 5-yr probability-of-mortality (*PM*) data sets for Douglas-fir and western hemlock trees receiving a single thinning. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 49,256*		*NOb = 12,673*	
PM	0.0208	NA	0.0416	NA
D	6.49	0.2–68.7	6.30	0.2–32.0
PCR	0.67	0.14–0.97	0.67	0.24–1.00
BAL	57.0	0.0–390.4	109.4	0.0–352.8
Individual plot	*NOb = 755*		*NOb = 540*	
PLEN	5.05	3.0–7.0	5.29	3.0–7.0
nt	1.0	1.0–1.0	1.0	1.0–1.0
YT_1	2.7	0.0–20.0	2.6	0.0–20.0
$PREM_1$	0.389	0.009–0.922	0.352	0.009–0.888
BH AGE	29.6	8.0–79.0	32.9	6.2–77.0
BA	105.8	6.6–393.8	128.7	6.6–354.5
Installation	*NOb = 75*		*NOb = 72*	
SI_{SP}	105.8	63.0–156.0	97.6	58.1–124.8

"single fertilization" data, consisting of all measurements from plots that had been fertilized only once. *CR* was predicted by Eq. [1] for western hemlock and the equation of Zumrawi and Hann (1989) for Douglas-fir. The resulting data sets, including variables used in the final mortality rate equations, are described in Table 32.

Table 32. Description of the 5-yr probability-of-mortality (*PM*) data sets for Douglas-fir and western hemlock trees receiving a single fertilization. The means were computed from the number of observations reported for each variable.

	Douglas-fir			Western hemlock	
Variable	Mean	Range		Mean	Range
Individual tree	*NOb = 76,534*			*NOb = 41,503*	
PM	0.0688	NA		0.1045	NA
D	7.34	0.6–32.7		5.84	0.6–30.7
PCR	0.46	0.15–0.95		0.49	0.16–0.99
BAL	117.6	0.0–409.7		167.2	0.0–411.7
Individual plot	*NOb = 1,510*			*NOb = 953*	
PLEN	5.44	3.0–7.0		5.54	3.0–6.1
nf	1.0	1.0–1.0		1.0	1.0–1.0
YF_1	3.2	0.0–16.0		2.9	0.0–16.0
PN_1	284.1	33.9–803.0		309.0	33.9–803.0
BH AGE	35.6	10.6–87.0		35.3	10.6–85.1
BA	197.7	9.6–412.3		221.8	18.9–412.3
Installation	*NOb = 197*			*NOb = 159*	
SI_{SP}	110.6	56.1–156.0	SI_{WH}	103.0	43.0–136.0

"single thinning and fertilization" data, consisting of all measurements from plots that had received one thinning and one fertilization. The thinning and fertilization did not have to occur at the same time. *CR* was predicted for all trees by Eq. [2]. The resulting data sets, including variables used in the final mortality rate equations, are described in Table 33.

Table 33. Description of the 5-yr probability-of-mortality (*PM*) data sets for Douglas-fir and western hemlock trees receiving a single thinning and a single fertilization. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 26,350*		*NOb = 20,564*	
PM	0.0318	NA	0.0643	NA
D	7.31	0.3–36.1	7.88	0.4–39.2
PCR	0.53	0.17–0.95	0.55	0.23–1.00
BAL	86.2	0.0–342.5	138.3	0.0–347.3
Individual plot	*NOb = 622*		*NOb = 660*	
PLEN	5.80	3.0–7.0	5.92	3.0–7.0
nt	1.0	1.0–1.0	1.0	1.0–1.0
YT_1	3.3	0.0–15.0	2.8	0.0–8.0
$PREM_1$	0.322	0.141–0.731	0.310	0.141–0.854
nf	1.0	1.0–1.0	1.0	1.0–1.0
YF_1	3.3	0.0–15.0	2.8	0.0–6.2
PN_1	332.9	200.0–803.0	359.2	200.0–803.0
BH AGE	40.5	10.6–74.2	46.9	10.6–74.2
BA	156.0	11.2–347.6	186.5	11.2–347.6
Installation	*NOb = 48*		*NOb = 51*	
SI_{SP}	100.2	60.0–156.0	95.5	55.4–122.0

"multiple thinning" data, consisting of all measurements from plots that had been thinned more than once. *CR* was predicted for all trees by Eq. [2]. The resulting data sets, including variables used in the final mortality rate equations, are described in Table 34.

Table 34. Description of the 5-yr probability-of-mortality (*PM*) data sets for Douglas-fir and western hemlock trees receiving multiple thinnings. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 65,644*		*NOb = 7,634*	
PM	0.0218	NA	0.0296	NA
D	11.07	1.2–59.2	7.04	1.5–23.0
PCR	0.56	0.19–0.95	0.75	0.35–1.00
BAL	80.8	0.0–297.7	98.5	0.0–263.9
Individual plot	*NOb = 1,596*		*NOb = 757*	
PLEN	4.74	3.0–7.0	4.57	3.0–7.0
nt	3.8	2.0–7.0	3.6	2.0–7.0
YT_1	2.1	0.0–20.0	2.1	0.0–19.0
$PREM_1$	0.157	0.007–0.646	0.151	0.007–0.578
BH AGE	33.7	11.0–81.0	31.5	11.0–81.0
BA	120.7	32.7–312.1	120.3	32.7–264.2
Installation	*NOb = 24*		*NOb = 18*	
SI_{SP}	123.0	85.8–156.0	107.3	78.6–124.8

"multiple fertilization" data, consisting of all measurements from plots that had been fertilized more than once. *CR* was predicted by Eq. [1] for western hemlock and by the equation of Zumrawi and Hann (1989) for Douglas-fir. The resulting data sets, including variables used in the final mortality rate equations, are described in Table 35.

Table 35. Description of the 5-yr probability-of-mortality (*PM*) data sets for Douglas-fir and western hemlock trees receiving multiple fertilizations. The means were computed from the number of observations reported for each variable.

	Douglas-fir		Western hemlock	
Variable	Mean	Range	Mean	Range
Individual tree	*NOb = 9,734*		*NOb = 3,577*	
PM	0.1091	NA	0.1770	NA
D	9.75	1.5–31.6	5.70	1.5–18.6
PCR	0.36	0.17–0.76	0.44	0.23–0.84
BAL	154.1	0.0–359.6	191.7	0.0–359.3
Individual plot	*NOb = 230*		*NOb = 107*	
PLEN	4.04	4.0–5.1	4.11	4.0–5.1
nf	2.42	2.0–3.0	2.42	2.0–3.0
YF_1	0.1	0.0–4.0	0.1	0.0–4.0
PN_1	200.0	100.0–400.0	210.4	100.0–400.0
BH AGE	40.7	23.0–59.0	38.3	23.0–57.0
BA	250.0	118.0–361.0	262.0	118.0–376.2
Installation	*NOb = 73*		*NOb = 40*	
SI_{SP}	117.0	70.0–151.0	109.3	64.3–132.7

Data Analysis and Results

The general approach taken to modeling 5-yr mortality rate was to develop a logistic equation for predicting *PM* of untreated trees and then to evaluate whether that equation could be applied to predict the *PM* in thinned and/or fertilized plots. If the equation for untreated plots was not applicable to a treatment type, it was modified to include the effects of treatment. The general logistic model form used in the analysis was Hamilton (1974):

$$PM = [1.0 + e^{-(Z_C + Z_F + Z_T + Z_{F\&T})}]^{-PLEN} \quad [19]$$

where

Z_C = mortality on untreated plots

Z_F = fertilization effects on mortality

Z_T = thinning effects on mortality

$Z_{F\&T}$ = fertilization and thinning effects on mortality

$PLEN$ = length of the growth period in 5-yr increments

= (length of the growth period in yr)/5

Equation for Untreated Plots

The following equation form for Z_C was determined by fitting the logistic Eq. [19] to the "untreated" data for both species (Table 30) with the weighted nonlinear program RISK (Hamilton 1974):

$$Z_C = c_0 + c_1 D + c_2 PCR_{VER} + c_3 SI_{SP} + c_4 BAL \qquad [20]$$

where

PCR_{VER} = predicted CR from HCB equation "VER"

VER = SMC for equations developed in this study

= Z & H for equation of Zumrawi and Hann (1989)

Table 36. Parameters for predicting the 5-yr probability of mortality (*PM*) for untreated and thinned Douglas-fir and western hemlock, Eq. [20] inserted into Eq. [19].

Parameter	Douglas-fir	Western hemlock
c_0	-3.27180	-0.761609
c_1	-0.381656	-0.529366
c_2	-2.98006	-4.74019
c_3	0.0182393	0.0119587
c_4	0.0112023	0.00756365

This equation form for Z_C has been used to model *PM* of predominantly untreated stands in SWO-ORGANON (Hann and Wang 1990). Numerous alternative formulations were evaluated, but none proved superior to this formulation. The resulting parameter estimates are given in Table 36.

The signs and magnitudes of the parameters were reasonable for both species when compared with previous studies (e.g., Hann and Wang 1990). Summary tables were prepared to examine how well the equations fit the data across the following attributes: predicted probability of survival (*PS*, which is 1.0 - *PM*), *D*, PCR_{VER}, SI_{SP}, and *BAL*. Each attribute of interest was divided into size classes, and the actual and the predicted number of trees surviving in each class were determined. The following statistics were then computed for each class:

- the difference of predicted survival rate minus actual survival rate, expressed as a percentage of the actual survival rate (the "% difference")
- the difference of predicted survival rate minus actual survival squared and then divided by predicted survival rate (the "chi-squared value").

Because *PM* is often very small, *PS* was used in the summary tables to avoid problems of excessively large statistics caused by dividing by values near 0.

Perfect predictions would result in values = 0 for both statistics across all classes in the summary tables. When the predictive equation is less than perfect, it is desirable that the percent differences not be always positive or negative or that there be no long runs of positive or negative values across the classes of an attribute, indicating a trend not explained by the attribute in the equation.

A chi-squared "lack of fit" statistic can also be formed by summing the chi-squared values across all classes, and a significance test can be formed by comparing this "lack of fit" statistic against a critical chi-squared value. Examination of these values and test statistics for the two species indicated that the equations predicted *PM* well in untreated plots.

Equation for a Single Thinning

The effect of a single thinning on *PM* was evaluated by using PCR_{SMC} to predict the *PM* and *PS* with Eqs. [19] and [20] for untreated plots and then preparing the previously described summary tables across *PS*, *D*, PCR_{SMC}, SI_{SP}, and *BAL* for each of 12 subsets of the single-thinning data. These 12 subsets were formed by first dividing the overall data sets (Table 31) into three classes by $PREM_1$ (0.0–0.33, 0.34–0.66, ≥0.67) and then further dividing each of these three classes into four additional classes by YT_1 (0–3, 4–7, 8–11, 12–15). Examination of these tables and the associated chi-squared test statistics indicated that the *PM* on thinned plots was adequately predicted by the combination of Eqs. [19] and [20] for untreated plots.

Equation for a Single Fertilization

The effect of a single fertilization on *PM* was evaluated by using PCR_{VER} to predict *PM* and *PS* with untreated-plot Eqs. [19] and [20] and then preparing the previously described summary tables across *PS*, *D*, PCR_{VER}, SI_{SP}, and *BAL* for each of 10 subsets of the single fertilization data. These 10 subsets were formed by first dividing the overall data sets (Table 32) into two classes by PN_1 (200 and 400) and then further dividing each of these two classes into five additional classes by YF_1 (0–3, 4–7, 8–11, 12–15, 16–20).

Examination of these tables and the chi-squared test statistics for western hemlock indicated that the *PM* on fertilized plots was adequately predicted by the combination of Eqs. [19] and [20] for untreated plots. In the case of Douglas-fir, however, the *PM* on fertilized plots was not adequately predicted by the combination of Eqs. [19] and [20] for untreated plots. After examining a number of alternatives, we found that the effect of a single fertilization (Z_{SF}) can be predicted by

$$Z_{SF} = c_5 PN_1^{1.5} e^{-0.5YF_1} \qquad [21]$$

The resulting parameter estimate is given in Table 37.

Table 37. Parameter for predicting the fertilization response of 5-yr probability of mortality (*PM*) for Douglas-fir and western hemlock, Eqs. [21] and [22] inserted into Eq. [19].

Parameter	Douglas-fir	Western hemlock
c_5	0.0000552859	0.0

Equation for a Single Thinning Combined with a Single Fertilization

Because the mortality rate of a single thinning was the same as that predicted by the untreated-plot equations (Eqs. [19] and [20]), we assumed that the effect of a single thinning combined with a single fertilization on 5-yr mortality rate of Douglas-fir could be adequately modeled by the equations for a single fertilization (Eqs. [19], [20], and [21]).

Equation for Multiple Thinnings

Because a single thinning had no additional impact on the mortality rate and could be characterized by the combination of Eqs. [19] and [20], the effect of multiple thinnings on 5-yr mortality rate of Douglas-fir and western hemlock was also modeled by the combination of Eqs. [19] and [20].

Equation for Multiple Fertilizations

Equation [21] predicts that the effect of a single fertilization on *PM* declines exponentially as YF_1 increases. Therefore, one way to characterize multiple fertilizations would be to "discount" the *PN*s in more distant fertilizations forward to the time of the most recent fertilization and to add these discounted *PN*s to PN_1. Mathematically, the discounted *PN* for the i^th fertilization would be computed by

$$\text{Discounted } PN_i = PN_i e^{-0.3333(YF_i - YF_1)}$$

The effect of one or more fertilizations (Z_F) would, therefore, be predicted by

$$Z_F = c_5[PN_1 + \sum_{i=2}^{nf} PN_i e^{-0.3333(YF_i - YF_1)}]^{1.5} e^{-0.5YF_1} \quad [22]$$

The form of Eq. [22] has been structured to reduce to the form of Eq. [21] when only one fertilization is applied. The effect of multiple fertilizations on PM is, therefore, predicted by the combination of Eqs. [19] and [22] with the parameter estimate in Table 37.

Equation for Multiple Thinnings Combined with Multiple Fertilizations

Because we assumed that the mortality rate from multiple thinnings was the same as that predicted by the untreated-plot equations (Eqs. [19] and [20]), we further assumed that the effect of multiple thinnings combined with multiple fertilizations on 5-yr mortality rate of Douglas-fir could be adequately modeled by the equations for multiple fertilizations (Eqs. [19], [20], and [22]).

Discussion

Combined Eqs. [19] and [20] for untreated plots and the associated parameter estimates for each species in Table 36 predict a decrease in *PM* with an increase in *D* and *CR* and an increase in *PM* with an increase in SI_{SP} and *BAL*. These results agree with those of Hann and Wang (1990).

The analysis could not detect an additional influence of thinning on the *PM*. Apparently the tree and plot attributes in the combination of Eqs. [19] and [20] were adequate to characterize the mortality rates after thinning. This implies that the thinnings applied on these experimental plots were conducted carefully and, therefore, avoided logging damage.

The analysis did detect an additional influence of fertilization on the *PM* of Douglas-fir. The combination of Eqs. [19] and either [21] or [22] (and their associated parameter in Table 37) predicts an increase in *PM* after fertilization. The predicted *PM* increases with an increase in *PN* and decreases with an increase in *YF*. Miller et al. (1986) and Shen et al. (2001) also observed that the mortality rate of Douglas-fir increased after fertilization, with the increase being greater on plots treated with more *PN*.

Maximum Size-Density Lines and Trajectories

The maximum size-density lines and associated trajectories for approaching the lines are used in ORGANON to constrain predicted maximum densities to reasonable values (Hann and Wang 1990). In general, maximum size-density concepts are based on the observation that stands approach a limit over time that defines maximum average size per tree in stands of a given density (Reineke 1933; Yoda et al. 1963; Drew and Flewelling 1977, 1979). This limit has often been characterized by the following maximum size-density line (with QMD as the measure of maximum average size per tree):

$$MLQ_i = g_1 + g_2 LT_i \qquad [23]$$

where

MLQ_i = natural log of maximum QMD at the i^{th} measurement for a given number of trees per ac

LT_i = natural log of number of trees per ac at the i^{th} measurement

Smith and Hann (1984, 1986), Puettmann et al. (1992), and Puettmann et al. (1993) then developed the following equation to characterize the approach of a stand to its maximum size-density line (i.e., the maximum size-density trajectory):

$$LQ_i = MLQ_i - (g_1 + g_2 LT_0 - LQ_0)e^{-g_3(LT_0 - LT_i)} \qquad [24]$$

where

LQ_i = natural log of QMD at the i^{th} measurement

LT_0 = natural log of initial number of trees per ac when mortality starts

LQ_0 = natural log of QMD when mortality starts

Equation [24] can be modified for the common situation when the first measurements of N and QMD are not equal to the initial planting density and initial size immediately before the start of self-thinning (i.e., the first measurements were taken later in the development of the plot):

$$LQ_i = MLQ_i - \{\frac{(g_1 g_4)^2}{(g_1 + g_2 LT_1 - LQ_1)}\}e^{-g_3(LT_1 - LT_i)} \qquad [25]$$

where

LT_1 = natural log of the number of trees per ac for the first measurement on the plot

LQ_1 = natural log of *QMD* for the first measurement on the plot

g_4 = a regression parameter that is the natural logarithm of the relative density for the initiation of mortality

This modification assumes that the initiation of mortality occurs on a line that parallels the maximum size-density line and that LT_1 and LQ_1 fall on the size-density trajectory. The modification is most effective when the first measurements are made near the initialization of mortality. The parameter values derived from the fit of Eq. [25] to the data can then be used in Eq. [24] to predict how a plot will approach the maximum size-density line.

The single-tree *PM* equations can be combined with the maximum size-density trajectory equation by the following approach:

(1) The single-tree *PMs* are first computed by Eq. [19], with $PLEN = 1.0$.

(2) These *PMs* are used to compute the number of trees and *QMD* of the plot at the end of the growth period.

(3) If the resulting *QMD* is ≤*QMD* predicted from the size-density trajectory of the plot (Eq. [24]), nothing further is done.

(4) If the *QMD* of the plot at the end of the growth period is >*QMD* predicted from the size-density trajectory of the plot (Eq. [24]), the *PMs* from Eq. [19] are increased as necessary to restore the plot to the size-density trajectory.

The combined *PM* can be expressed as

$$CPM = [1.0+e^{-(KR + Z_C + Z_F)}]^{-1} \quad [26]$$

where

CPM = the combined *PM*

KR = coefficient to correct Eq. [19] and place the plot on the size-density trajectory of Eq. [24] when the *QMD* of the plot at the end of the growth period is greater than that predicted from the size-density trajectory of the plot (Eq. [24]).

The value of *KR* is determined by Reineke's (1933) relative density (RD_R) of the plot at the start of the growth period and by the gross 5-yr basal area growth of the plot (Hann and Wang 1990). If RD_R at the start of the growth period is <0.6 [where competition-induced mortality should start (Long 1985)] or if no correction is needed to place the ending *QMD* of the plot on the size-density trajectory, *KR* is set to 0.0 and mortality rate is predicted from Eq. [19]. If a correction is needed and 0.6 < initial RD_R < 1.0, different values for *KR* are systematically substituted into Eq. [26] until the number of trees and *QMD* fall on the size-density trajectory defined by Eq. [24]. If a correction is needed and the starting RD_R is >1.0, Eq. [26] is solved iteratively with various values of *KR* until *N* and *QMD* at the end of the growth period fall on the maximum size-density line defined by Eq. [23]. Finally, if the stand is thinned at the start of the growth period and RD_R at that time is >0.6, *KR* is set to 0.0 for all subsequent growth periods until the size-density trajectory of the thinned stand again equals or exceeds that of the unthinned stand. Eq. [26] then is again solved iteratively in order to find a *KR* that will keep the stand on the size-density trajectory.

This approach differs from that described in Hann and Wang (1990), which multiplied $(Z_C + Z_F)$ by *KR*. Subsequent testing of their approach indicated that it caused a reduction (instead of the intended increase) in *CPM* for trees with a *PM* > 0.5. Our new approach guarantees that *CPM* will always be $\geq$*PM*.

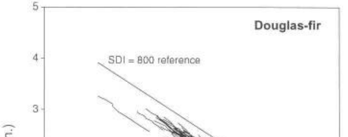

Figure 5. Maximum size-density diagram of the 128 Douglas-fir plots and 39 western hemlock plots exhibiting straight-line segments in the size-density relationship.

Data Description

Three data sets were created for analysis of maximum-size density lines and their trajectories. The first data set consisted of QMD_i and N_i measurements from long-term untreated plots in which QMD_1 and N_1 were judged to be close to QMD_0 and N_0. Screening the data showed nine Douglas-fir installations and no western hemlock installations with control plots that met this criterion. Seven of the nine installations came from the LOGS cooperative: Sayward and Shawnigan Lake in British Columbia; Clemons, Francis, and Iron Creek in Washington; and Hoskins and Stampede Creek in Oregon. The eighth installation was the spacing study at Wind River Experiment Forest in Washington (Reukema 1979), and the ninth was the Lookout Mountain installation in Washington (data provided by the Pacific Northwest Research Station, USDA Forest Service). For each installation, the control plots were merged and QMD_i and N_i were computed for each measurement.

The second data set was developed by ocularly screening graphs of LQ_i over LT_i for control plots and choosing those control plots (or portions of the data for a control plot) in which the graph of LQ_i over LT_i for the most recent measurements on each plot was a straight-line segment composed of at least three measurements. The control plots used to create the first data set were also screened. This process identified 128 untreated Douglas-fir plots and 39 western hemlock plots with straight-line segments (Figure 5); 26 of the Douglas-fir plots were also used in creation of the first data set.

The third data set was also developed by ocularly screening graphs of LQ_i over LT_i for fertilized Douglas-fir plots and choosing those fertilized plots (or portions of the data for a fertilized plot) in which the graph of LQ_i over LT_i for the most recent measurements on each plot was a straight-line segment composed of at least three measurements. The Douglas-fir plots screened for this data set came from a subset of the installations chosen for the second data set; only those installations that met the following criteria were screened for creation of the third data set:

- the installation had both control and fertilized plots
- the slope of the straight-line segment for the control plots on the installation was not significantly different from -0.62305, the reciprocal of Reineke's (1933) slope of -1.605, with $P = 0.05$.

This process identified 1,199 measurements of QMD_i and N_i on 86 control and 148 fertilized Douglas-fir plots from 43 installations. Thirty-seven of the fertilized plots on these installations had received a single fertilization of $PN = 200$, 63 had received multiple fertilizations of $PN = 200$, 31 had received a single fertilization of $PN = 400$, and 17 had received a single fertilization of $PN = 400$ and one or more additional fertilizations of $PN = 200$.

Data Analysis and Results

Control Plots

Nonlinear regression analysis was used to fit a modified version of Eq. [25] to the data from the long-term untreated plots in which QMD_I and N_I were judged to be close to QMD_0 and N_0. Eq. [25] was modified to allow the evaluation of potential differences between installations in the intercept parameter, g_1, through the use of indicator variables:

$$LQ_i = [(g_1 + \sum_{j=1}^{8} g_{1,j} I_j) + g_2 LT_i] - \{\frac{[(g_1 + \sum_{j=1}^{8} g_{1,j} I_j) g_4]^2}{[(g_1 + \sum_{j=1}^{8} g_{1,j} I_j) + g_2 LT_1 - LQ_1]}\} e^{-g_3(LT_i - LT_1)} \quad [27]$$

where

$g_{1,j}$ = a regression intercept adjustment parameter for the j^{th} installation, $j = 1$ to 8

I_j = an indicator variable for the j^{th} installation

= 1.0 if the data came from the j^{th} installation

= 0.0 otherwise

In this formulation, g_1 was the intercept parameter for the Hoskins installation, and the regression parameters on the indicator variables indicated installation-specific adjustments to g_1.

The resulting slope parameter, g_2, was significantly >-0.5 ($P = 0.05$). A slope of -0.5 for the maximum size-density line produces a constant *BA* as *LQ* of a plot moves up the line, a slope value < -0.5 [such as Reineke's (1933) slope value of -0.62305] produces an increasing *BA* as the *LQ* moves up the line, and a slope value >-0.5 produces a decreasing *BA* as the *LQ* moves up the line. This last behavior would not be expected under normal self-thinning. Examination of the data from the individual installations showed that the data from the Wind River spacing trial were causing the slope to be >-0.5. The closer spacings at the Wind River spacing trial had experienced substantial snow- and ice-caused mortality in clumps throughout the plots (Reukema 1979). Because this pattern is atypical of competition-induced mortality, the Wind River data were removed from further analysis.

Table 38. Parameters and asymptotic standard errors for predicting the maximum size-density line and its trajectory for control plots from eight Douglas-fir installations, Eq. [27].

Parameter/ Standard error	Douglas-fir
g_1	6.26729808
SE(g_1)	(0.0125261573)
$g_{1,1}$	-0.09103427
SE($g_{1,1}$)	(0.0248646716)
$g_{1,2}$	-0.18731227
SE($g_{1,2}$)	(0.0270601852)
$g_{1,3}$	-0.14199770
SE($g_{1,3}$)	(0.0237004595)
$g_{1,4}$	-0.25923972
SE($g_{1,4}$)	(0.0222662694)
$g_{1,5}$	-0.31842013
SE($g_{1,5}$)	(0.0239275067)
g_2	-0.62305
SE(g_2)	(NA)
g_3	-14.39533971
SE(g_3)	(3.2568460886)
g_4	-0.51082562
SE(g_4)	(NA)

NA: Not applicable

A refit of Eq. [27] to the reduced data set produced a slope parameter, g_2, that was not significantly different from the reciprocal of Reineke's slope (i.e., -0.62305) at $P = 0.05$. Therefore, the slope value was set to -0.62305 and the remaining parameters were reestimated. The resulting installation indicator variables on g_1 were examined for significant difference from 0 ($P = 0.05$). Two of the installations (Francis and Iron Creek) had intercept adjustments that met this criterion. They were combined with the Hoskins installation, represented by the overall intercept value, and the parameters were reestimated. This resulted in five intercept corrections for the following installations:

j = 1 for data from Sayward
= 2 for data from Shawnigan Lake
= 3 for data from Stampede Creek
= 4 for data from Lookout Mountain
= 5 for data from the Clemons installation

The parameter estimates and their SEs are in Table 38.

Because the intercept adjustments for these installations were all negative, their intercept values were significantly smaller than the other three. As a result, predicted maximum *SDI* values ranged from 348 to 580 on the eight installations, with an average of 483. Also, the parameter value for g_4 was not significantly different at $P = 0.05$ from the natural log of Long's (1985) RD_R for the onset of competition-induced mortality (i.e., $RD_R = 0.6$).

To verify these results, we fit simple linear equations by linear regression to each of the 128 straight-line segments from Douglas-fir plots and 39 straight-line segments from western hemlock plots that formed the second data set. A *t*-test was used to determine if the slope parameter for each regression analysis was significantly different from the reciprocal of Reineke's (1933) slope value of -1.605 (i.e., -0.62305), with $P = 0.01$. Slopes of 27 of the 128 Douglas-fir regressions (21%) and 4 of the 39 western hemlock regressions (10%) differed significantly from the reciprocal of Reineke's slope. The slopes for the other 101 Douglas-fir straight-line segments and 35 western hemlock straight-line segments were then set to -0.62305 and the intercept terms were recalculated. The resulting maximum *SDI* values for this subset of the Douglas-fir data ranged from 268 to 657 (average, 454), confirming that more than one maximum *SDI* value is applicable on Douglas-fir plots, as found in the first analysis. The maximum *SDI* values for the subset of the western hemlock data ranged from 467 to 783 (average, 590).

Table 39. Parameters and asymptotic standard errors for predicting the maximum size-density line and its trajectory for control plots from eight Douglas-fir installations, Eq. [25]. Parameters g_2 and g_4 were fixed to values from Reineke (1933) and Long (1985), respectively.

Parameter/ Standard error	Douglas-fir
g_1	6.16819645
SE(g_1)	(0.018874713)
g_2	-0.62305
SE(g_2)	(NA)
g_3	-22.05958933
SE(g_3)	(10.843313780)
g_4	-0.51082562
SE(g_4)	(NA)

NA: Not applicable.

In order for maximum *SDI* to be applicable in the ORGANON model, however, there must be some mechanism for predicting which maximum *SDI* value is appropriate for a given plot or stand. To explore if maximum *SDI* is predictable from available attributes, we used data from the 101 Douglas-fir plots with straight-line segments that followed the reciprocal of Reineke's (1933) slope to produce graphs of maximum *SDI* plotted across site index, latitude, % basal area in Douglas-fir, and stand origin (natural, plantation, or unknown) of each plot. These graphs showed no trends. We therefore concluded that it was not possible to develop a method for predicting maximum *SDI* from the available plot and installation attributes.

Eq. [25] was then fit to the data from the eight installations with nearly complete trajectories. In this fit, g_2 was fixed to -0.62305 and g_4 was fixed to the natural log of 0.6 (i.e., -0.51082562), and the remaining parameters in Eq. [22] were estimated by nonlinear regression. The resulting parameters and their SEs are in Table 39.

Fertilized Plots

Limitations of the data sets available for modeling restricted our ability to comprehensively evaluate the potential effect of fertilization on the maximum size-density line and trajectory. To evaluate whether the intercept term of the maximum size-density line was affected by fertilization, we fit the following equation to the third data set by linear regression:

$$LQ_1 + 0.62305[LT_1] = h_0 + h_1I_1 + h_2I_2 + h_3I_3 + h_4I_4 \quad [28]$$

where

I_1 = 1.0 if the plot had received a single fertilization of $PN = 200$

Table 40. Parameters and asymptotic standard errors for evaluating whether fertilization affects the intercept term of the maximum size-density line for Douglas-fir installations, Eq. [28].

Parameter/ Standard error	Douglas-fir
h_0	6.109008
SE(h_0)	(0.00599914)
h_1	0.002055
SE(h_1)	(0.00885292)
h_2	0.021049
SE(h_2)	(0.00817829)
h_3	-0.002847
SE(h_3)	(0.00914860)
h_4	-0.002670
SE(h_4)	(0.01086490)

= 0.0 otherwise

I_2 = 1.0 if the plot had received multiple fertilizations of PN = 200

= 0.0 otherwise

I_3 = 1.0 if the plot had received a single fertilization of PN = 400

= 0.0 otherwise

I_4 = 1.0 if the plot had received a single fertilization of PN = 400 and one or more additional fertilizations of PN = 200

= 0.0 otherwise

This analysis assumes that the slope of the relationship is adequately portrayed by the reciprocal of Reineke's (1933) slope value. The resulting parameters and their SEs for Eq. [28] are in Table 40.

Discussion

The results of fitting Eq. [27] to the Douglas-fir maximum size-density trajectory data and of fitting simple linear equations to the 128 Douglas-fir and 39 western hemlock maximum size-density line segments strongly suggest that neither Douglas-fir nor western hemlock plots approach a single maximum *SDI* value as they develop. As a consequence, the potential yield for a given site depends not only on the *SI* of the plot, but also on its maximum *SDI*. Density-related differences in potential yield have been found for numerous other tree species, including loblolly pine (Hasenauer et al. 1994). Hasenauer et al. (1994) reported that maximum *SDI* of loblolly pine varied from region to region across its distribution and hypothesized that possible causes could be differences in genetics, soils, or other factors. We were unable to detect trends in maximum *SDI* across site index, latitude, species composition, or stand origin from the data available. A better understanding of these differences may be found when the SMC Type III planting density studies (Chappell and Osawa 1991) have matured.

None of the fertilization parameters of Eq. [28] (Table 40) differed significantly from 0 at $P = 0.05$, indicating that fertilization does not affect the intercept of the maximum size-density line for Douglas-fir. This agrees with the work of White and Harper (1970) and Smith and Hann (1984), who found that site quality does not affect the configuration of the maximum size-density line or trajectory. Rather, site quality influences the growth rate of the stand, and, as a result, how fast a stand moves along the trajectory. Stands with high growth rates (e.g., high site quality) move along the trajectory and associated maximum size-density line faster than do stands with low growth rates (e.g., low site quality). This behavior has been called the Suchatschew effect (Harper 1977). As a result of the Suchatschew effect, mortality rate increases with increasing site quality. Fertilization of unthinned Douglas-fir increases the rate of mortality, indicating that fertilization causes the Suchatschew effect, rather than changing the configuration of the maximum size-density trajectory (Miller 1981; Miller et al. 1986).

Integrating the Equations into ORGANON

The *HCB*, ΔD, ΔH, *PM*, and maximum size-density and trajectory equations reported in this publication and the *H-D* equations of Hanus et al. (1999) were inserted into the SMC version of ORGANON (SMC-ORGANON). An extensive verification was then conducted to ascertain that all of the equations and parameters had been correctly entered into the software.

Once verification was completed, the predictive behavior of the model was evaluated by the SMC Modeling Project using the control plots of the LOGS studies. The LOGS studies were chosen for the evaluation because of their relatively long series of remeasurements. This evaluation proceeded as follows:

(1) Data from the first measurement on each of the LOGS control plots were read into SMC-ORGANON, and missing *H*s and *HCB*s were calculated.

(2) The completed tree lists were then used to project stand development for the total duration of measurements available on the various LOGS control plots.

(3) Predictions of *N*, *BA*, and *H40* from SMC-ORGANON after each 5-yr growth period were compared to the actual measurements on each of the LOGS plots.

These comparisons indicated the following behavioral problems:

- The *PM*s were too low after the plots had entered the zone of competition-induced mortality.
- The basal area growth rates were too high, indicating overpredictions from the ΔD equations.

The first problem was attributed to the value for the parameter g_3 in the size-density trajectory portion of Eq. [24]. In forcing a single intercept value in the maximum size-density line and trajectory, the value of g_3 changed from -14.39533971 for Eq. [27] with multiple intercept values (Table 38) to -22.05958933 for Eq. [24] with a single intercept (Table 39). We decided that Eq. [24] was a misspecification of the underlying model form characterizing the maximum size-density line and trajectory data available to us, so g_3, as well as g_2 and g_4, was set to the value of Eq. [27]. Parameter g_1 was set to 6.19958 (which corresponds to a maximum *SDI* of 520.5), the value used in the northwest Oregon version of ORGANON. Because of our finding that maximum *SDI* can change between stands, however, we have added a feature to the new edition of ORGANON that allows advanced users to set their own maximum *SDI* values for a given stand. Deciding the appropriate maximum *SDI* value to use for a given stand is, of course, the biggest challenge in applying this approach. We do know from tests using the new edition of ORGANON that changing the maximum *SDI* for a stand can impact the predicted yield of the stand as much as many treatment schemes.

The second problem we attributed to the underestimation of *HCB* for trees without measurements of *HCB*. As mentioned above, comparing predicted *HCB* from Eq. [1] to that predicted from the *HCB* equation of Zumrawi and Hann (1989) for Douglas-fir showed substantially larger crowns being predicted from Eq. [1]. When the Zumrawi and Hann (1989) equations were used to fill in missing values, the problems associated with the overprediction of ΔDs disappeared. Following the strategy suggested by Hanus et al. (2000), we decided to use the equation of Zumrawi and Hann (1989) to fill in missing values in the new edition of ORGANON and to use Eq. [1] of this study to predict change in *HCB*.

After completion of the verification within the SMC Modeling Project, SMC-ORGANON was released to the cooperators for their evaluation. Several of the cooperators raised concerns that the predicted ΔD response after fertilization was too small. Re-evaluation of the fertilization data for ΔD indicated that the approximately one-third of the data collected in Canada displayed a substantially different response to fertilization than did the data collected in the USA. Further examination of the Canadian data found the following:

- Over one-half of the data came from one installation established at Shawnigan Lake by Forestry Canada, and the *SI* calculated for this installation was incorrect. In our first analysis, the Shawnigan Lake installation had been treated mistakenly as two separate installations because the plots had been installed in two consecutive years. As a result, substantially different *SI*s were computed for the two parts of the installation (100.7 for the first year and 78.1 for the second), even though the plots for the two years were spatially intermixed. The underestimation of *SI* for the second-year data caused the underestimation discussed above that we attempted to adjust for in Eq. [8]. When the *SI* problem was fixed, the relatively large fertilization response became relatively small when compared with the USA data sets. Because of the well-documented variation in response to fertilization between installations (e.g., Opalach and Peterson 1986; Miller et al. 1986), allowing such a large proportion of the data to come from a single installation could distort the average that might be expected for the region.

- The data supplied by the British Columbia Ministry of Forests had very few observations in the growth period immediately following fertilization (the period where the largest response occurs) that could be used in the analysis. This problem arose because of the lack of height measurements on many of the BC installations immediately after treatment. Apparently, many of the installations were established and *D*s and *H*s measured before the start of the growing season, but the treatments were not applied until after its end, at which time all *D*s, but only a few *H*s, were remeasured.

Table 41. Parameters and asymptotic standard errors for predicting the response of 5-yr diameter-growth rate (ΔD) to fertilization in Douglas-fir and western hemlock, Eqs. [7] and [11], with the modeling data restricted to installations in the United States.

Parameter/ Standard error	Douglas-fir	Western hemlock
a_{10}	1.452150723	0.0
SE(a_{10})	(0.229647321)	(NA)
a_{11}	0.782839240	0.0
SE(a_{11})	(0.107882422)	(NA)
a_{12}	-0.234091974	0.0
SE(a_{12})	(0.044340580)	(NA)
a_{13}	-1.108430496	0.0
SE(a_{13})	(0.100689399)	(NA)

NA: Not applicable.

Because of these problems, we refitted Eqs. [7], [11], [14], and [15] using just the tree response data from the USA installations. The resulting parameters and their SEs for Eqs. [7] and [11] are found in Table 41. The revised fertilization response for ΔD was larger on low and medium SI than the response predicted in the original equation (Figure 6), indicating that the concern expressed by the cooperators was probably justified. In concordance with the findings of Curtis et al. (1981) and Miller et al. (1988), the revised fertilization response equation also predicts that the response increases at a decreasing rate with PN. Therefore, parameters for the ΔD response to fertilization equation for Douglas-fir in SMC-ORGANON were changed to the values reported in Table 41.

The refit Eqs. [14] and [15] also required the refit of the $C\Delta H40_C$ correction equation to the data from the USA alone:

$$C\Delta H40_C = (\Delta H40_C)(0.990883266 + 0.431894954e^{-[0.009433757(SI_{DF}-3)]})$$

The corrected fertilization data were then refit to Eq. [14] (and, as a result, Eq. [15]) by nonlinear regression. The resulting parameters and their SEs for Eqs. [14] and [15] are in Table 42. The revised fertilization response for ΔH is slightly larger than the response predicted by the original equation (Figure 7). Therefore, parameters for the ΔH response to fertilization equation for Douglas-fir in SMC-ORGANON were changed to the values in Table 42.

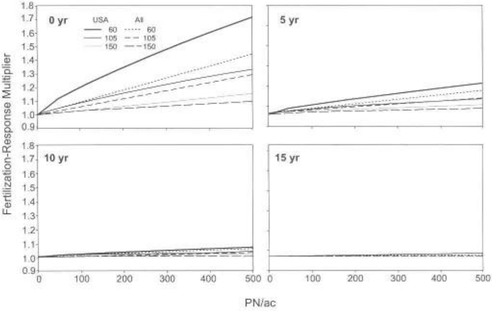

Figure 6. Revised Douglas-fir fertilization-response multiplier for 5-yr diameter growth rate (ΔD) 0, 5, 10, and 15 yr after fertilization.

Table 42. Parameters and asymptotic standard errors for predicting the potential 5-yr height-growth rate (ΔH) of fertilized Douglas-fir and western hemlock, Eqs. [14] and [15], with the modeling data restricted to installations in the United States.

Parameter/ Standard error	Douglas-fir	Western hemlock
b_1	-1.107409443	0.0
SE(b_1)	(30.225856216)	(NA)
b_2	-2.133334346	0.0
SE(b_2)	(0.159278078)	(NA)

NA: Not applicable.

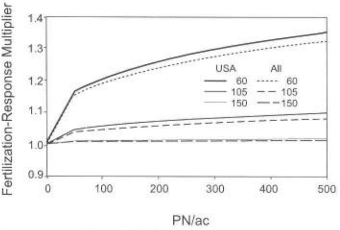

Figure 7. Revised Douglas-fir fertilization-response multiplier for 5-yr height growth rate (ΔH).

Finally, a review of the data sets used to develop the thinning-response modifier Eqs. [10] and [18] revealed that 94.9% of the Douglas-fir thinning data came from plots with a Reineke's relative density (RD_R, calculated using a maximum SDI of 520) ≥0.4 just before thinning. A question, therefore, arose about the applicability of the thinning-response modifiers to plots with $RD_R < 0.4$, most of which would have trees with much longer before-thinning crowns than those in the modeling data sets. Long (1985) developed a stand-density management diagram using RD_R that placed crown closure at $RD_R = 0.25$ and the lower limit to "full-site occupancy" at $RD_R = 0.35$. We decided to follow a conservative approach by reducing the thinning-response modifiers from their maximum effect at a $RD_R = 0.4$ before thinning to no effect (i.e., a value of 1.0 for $TR_{\Delta D}$ and $TR_{\Delta H}$) when RD_R before thinning fell below 0.25. The resulting constrained thinning-response modifier for ΔD was

$$TR_{\Delta D} = 1.0 + a_8(PREM_{\Delta D})(e^{a_9 YT_1})(RD_R MOD) \qquad [29]$$

where

$$RD_R MOD = e^{-[1.4(1-RD_R)]^{30}}$$

$$RD_R = \frac{SDI_{Disc}}{SDI_{Max}}$$

SDI_{Max} = Maximum Reineke's (1933) SDI for the species

SDI_{Disc} = Discounted Reineke's (1933) SDI

$$SDI_{Disc} = N_{Disc}\left(\frac{10}{QMD_{Disc}}\right)^{-1.605}$$

If $SDI_{Disc} > SDI_{Max}$, then $SDI_{Disc} = SDI_{Max}$

$$QMD_{Disc} = \sqrt{\frac{BA_{Disc}}{0.005454154(N_{Disc})}}$$

$$BA_{Disc} = BABT + \sum_{i=2}^{nt} BAR_i e^{a_9(YT_i - YT_1)}$$

$$N_{Disc} = NBT + \sum_{i=2}^{nt} NR_i e^{a_9(YT_i - YT_1)}$$

The resulting constrained thinning response modifier for ΔH was

$$TR_{\Delta H} = 1.0 + b_9(PREM_{\Delta H})^{b_{10}}(e^{b_{11}YT_i})(RD_R MOD) \quad [30]$$

where

$$BA_{Disc} = BABT + \sum_{i=2}^{nt} BAR_i e^{\frac{b_{11}}{b_{10}}(YT_i - YT_1)}$$

$$N_{Disc} = NBT + \sum_{i=2}^{nt} NR_i e^{\frac{b_{11}}{b_{10}}(YT_i - YT_1)}$$

The $RD_R MOD$ is sigmoidal over the range 0 to 1 for RD_R, and it predicts a value of 0.013 for $RD_R = 0.25$ and a value of 0.9945 for $RD_R = 0.40$.

Limitations of the Data

The primary strength of the existing permanent-plot data made available to the SMC Modeling Project by the cooperators was the large quantity of the data. In general, however, most of the data were of poor quality for developing single-tree stand- development models. Problems included the following:

- The data sets had too few measurements of H. Often, only enough heights were measured to define $H40$ for the plot. As a result, the sample was concentrated in undamaged, dominant trees. Hann and Ritchie (1988), Ritchie and Hann (1990), and the findings of this study have shown that ΔH, and therefore H, is affected by the position of the tree in the stand, and Hanus et al. (1999) have shown that damaging agents can also affect H.

- Many of the data sets had no measurements of *CR* and *CW*. When they were measured, too small a sample was taken, and often they were measured only on those trees where *H* was measured. As a result, their sample exhibited the same bias in sample selection as that of *H*. Ritchie and Hann (1987), Zumrawi and Hann (1989), Hanus et al. (2000), and the findings of this study have shown that *HCB* is affected by the position of the tree in the stand, and the work of Hanus et al. (2000) have shown that damaging agents can also affect the *HCB* of a tree.

- Many landowners or forest managers had taken too little attention in conducting field checks, editing the resulting data, and performing other data-management practices that are required to assure quality data.

- The geographic distribution of the data was not balanced among British Columbia, Washington, and Oregon.

- The data did not cover many of the stand structures and treatment regimes of primary interest to the public land managers.

- The dominant height growth equations available at the time for Douglas-fir did not adequately characterize the $\Delta H40$ in very young plantations. As a result, estimates of SI were greatly inflated.

- The size and number of the plots were too small, too few, or both on most of the fertilization installations. As a result, *SI*s on these installations were often underestimated. This can inflate estimates of fertilization response, particularly in low *SI*s.

The lack of crown measurements in most of the fertilization data sets was a serious shortcoming. On the one data set with *CR* measurements, fertilization increased *CL* and, once the impact of *CL* was considered, fertilization negatively impacted ΔD. Given this controversial finding and the fact that the analysis was based on only one installation, we chose not to include the possible fertilization impact on *HCB* in the final equation. However, we strongly recommend that this issue be addressed again when additional fertilization data with *HCB* measurements become available.

The lack of competing vegetation measurements and Douglas-fir *CW* measurements made early measurements of ΔD, ΔH, and mortality rates on the SMC Type I plots of little value for developing traditional single-tree stand-development models. There was obvious variation in tree growth that was not explainable by *CR* because most trees had a *CR* near 1.0. The developers of the Regional Vegetation Management Model (Shula et al. 1998) found that *CW* of Douglas-fir was more effective than *CR* at characterizing the competitive impact of competing vegetation on the growth of very young trees.

Evaluation of the Modeling Methods

Proven model forms and parameter estimation techniques were used to model the equations used to predict the *HCB*, ΔD, ΔH, *PM*, and maximum size-density and trajectory lines for untreated stands. These performed as expected. Thinning and fertilization treatment effects were modeled as either additions to (*HCB* and *PM*) or multipliers of (ΔD and ΔH) the basic equations for untreated stands. As a result of this approach, these treatment modifiers could also be used with the other versions of ORGANON or with other single-tree/distance-independent models with a structure similar to ORGANON.

The forms of the single-treatment modifiers were structured so as to give the responses to treatments expected from other studies. The multiple-treatment modifiers for ΔD and ΔH were structured so as to guarantee that the application of the single treatment in the general multiple-application response equation produced a prediction identical to that from the single-treatment response equation; that multiple applications spaced very close together provided a prediction very close to a combined single application of the same total amount (e.g., two 200-lb applications of nitrogen spaced 1 wk apart should approximately produce the same response as a single 400-lb application of nitrogen); and that multiple applications spaced far apart behaved as single, independent applications.

In reviewing the modeling work, however, it became evident that perhaps better modeling methods could have been used at times in developing some of these equations. Additional time at the end of the project would have allowed more thorough synthesis, analysis, comparisons, and standardization or modification of the modeling approaches being used by the different members of the project. The following summarizes some of the improvements that could have been explored had the project continued:

- Recent work using data from young to old growth, even- to uneven-aged, and pure to mixed species stands in southwest Oregon found that the following model form better characterized ΔD than Equation [4]:

  $$\Delta D_C = e^{a_0 + a_1 X_9 + a_2 X_{10} + a_3 X_3 + a_4 X_4 + a_5 X_{11} + a_6 X_6} \quad [31]$$

  where

  $X_9 = \ln(D + 5.0)$

  $X_{10} = D$

  $X_{11} = BAL / \ln(D + 2.7)$

- When compared with Eq. [4], Eq. [31] fit to Douglas-fir in southwest Oregon predicted that (1) the maximum ΔD was smaller and peaked at a smaller D; (2) for trees with small values of *BAL* (i.e., dominant trees), those with D <12-in. or >55-in. had larger ΔDs, and trees with D between 12 and 55 in. had smaller ΔDs; and (3) for trees with large values of *BAL* (i.e., intermediate and suppressed trees), ΔD was larger for all Ds. Based on both Furnival's (1961) index of fit and residual analysis, Eq. [31] fit the data better than did Eq. [4].

- The method used to expand the ΔD modeling data sets by using both PCR_{SMC} and measured *CR* may not have been fully adequate for dealing with the measurement error such practice introduces. Perhaps a better approach would have been to fit the following expansion of Eq. [4] (or an equivalent expansion of Eq. [31]):

  $$\Delta D_C = e^{a_0 + \sum_{i=1}^{6} a_i X_i + \alpha_0 I + \sum_{i=1}^{6} \alpha_i I X_i}$$

  where

  I = 1.0 if the observation used PCR_{SMC}

  = 0.0 if the observation used *CR*

  A *t*-test could then have been used to determine if the "α" parameters differed significantly from 0. Any that did would have indicated that the use of PCR_{SMC} did create a problem with measurement error.

  With the method used in this study, the magnitude of the impact of the measurement error on predicting ΔD is unknown. However, the independent evaluations done by Greg Johnson[2] on the western hemlock data indicate that the impact may not be too severe.

- The thinning and fertilization response equations for ΔD and the thinning-response equation for ΔH were analyzed at the tree level because of the need to evaluate whether the impact of treatment varied across tree-level variables such as *BAL* or *CCH*. Even though no impact was found at the tree level, the final analyses incorporating just plot-level attributes were still conducted using the tree-level data sets. As a result, plots with many tree observations influenced the regression equation more than did those with few observations. The solution to this problem would have been to compute a mean tree response for each plot and use those values, weighted by the reciprocal of the SE of the mean, in developing the plot-level response equations.

- In developing the various equations, not enough care was taken to ensure that exactly the same data sets were used in all of the analyses. For example, the fertilization response equation for ΔD excluded *PN* values >450, while the fertilization response equation for ΔH included those values.

[2] 2000. Willamette Industries, unpublished report.

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Appendix: Summary of Abbreviations

Variable	Units	Explanation
BA	ft^2/ac	Plot basal area
BABT	ft^2/ac	Total plot basal area before the last thinning
BAL	ft^2/ac	Plot basal area in trees with *D* > that of the subject tree
BAR_i	ft^2/ac	Plot basal area removed in the i^{th} thinning
BA_T	ft^2/ac	Current plot basal area plus basal area removed in past thinnings, discounted by an exponential function of the number of years since the thinning occurred
CA	ft^2	Area of the crown, assuming a circle with a diameter of *CW*
CCFL	ft^2/ac	Plot crown competition factor in trees with *D* >that of the subject tree
$CCFLR_i$	ft^2/ac	Plot crown competition factor in trees with *D* >that of the subject tree removed in the i^{th} thinning
$CCFL_T$	ft^2/ac	Current plot crown competition factor in trees with *D* >that of the subject tree plus crown competition factor in trees with *D* >that of the subject tree removed in past thinnings, discounted by an exponential function of the number of yr since thinning
CCH	%	Percent crown closure at the top of the tree for the plot
$C\Delta H40_C$	ft	Corrected 5-yr change in *H40* on untreated plots
CL	ft	Length of the live crown (*H* - *HCB*)
CPM	none	The combined probability of mortality
CR	none	Live crown ratio (*CL:H*)
CW	ft	Crown diameter at *RH*
D	in.	Diameter at 4.5 ft above ground level (breast height)
D40	in.	The average *D* of the 40 largest diameter trees/ac
ΔD	in.	5-yr diameter increment
ΔD_C	in.	5-yr diameter increment for a tree growing on untreated lots
ΔD_{Cij}	in.	Measured 5-yr diameter increment for the i^{th} tree growing on all untreated plots in the j^{th} installation that included the treatment of interest
$\Delta DMOD_F$	none	*YF* modifier to the equation for fertilization response of diameter increment

$\Delta D_{MT,i,j}$	in.	Measured 5-yr diameter increment for the i^{th} tree growing on multiply thinned plots from the j^{th} installation
$\Delta D_{SF,i,j}$	in.	Measured 5-yr diameter increment for the i^{th} tree growing on singly fertilized plots from the j^{th} installation
$\Delta D_{ST\&SF,i,j}$	in.	Measured 5-yr diameter increment for the i^{th} tree growing on singly thinned and singly fertilized plots from the j^{th} installation
$\Delta D_{ST,i,j}$	in.	Measured 5-yr diameter increment for the i^{th} tree growing on singly thinned plots from the j^{th} installation
ΔH	ft	5-yr height increment
$\Delta H40$	ft	5-yr change in the average height of the 40 largest diameter trees per ac
$\Delta H40_C$	ft	5-yr change in the average height of the 40 largest diameter trees/ac on an untreated plot
$\Delta H40_F$	ft	5-yr change in the average height of the 40 largest diameter trees/ac on a fertilized plot
$\Delta H40_T$	ft	5-yr change in the average height of the 40 largest diameter trees/ac on a thinned plot
ΔHCB	ft	5-yr change in height to the base of the live crown
$\Delta HMOD$	ft	5-yr height-growth modifier function
$\Delta HMOD_C$	ft	5-yr height-growth modifier function from trees on untreated plots
$\Delta HMOD_T$	ft	5-yr height-growth modifier function from trees on thinned plots
ΔH_T	ft	5-yr height increment of trees from thinned plots
EXPAN	no./ac	The number of trees/ac represented by the sampled tree
FERT	lb/ac	The total weight of nitrogen applied to the plot. Weight of nitrogen applied during former applications is discounted with an exponential function of the years since the application of the fertilizer
$FR_{\Delta D}$	none	Fertilization response modifier for 5-yr diameter increment combined across all fertilized plots on all installations
$FR_{\Delta H40}$	none	Fertilization response to the predicted 5-yr average dominant height growth equation combined across all fertilized plots on all installations
f_{SP}	ft	The *H40* function for species "SP"
GEA	yr	The age of a dominant tree with the same height on the same site as the subject tree
H	ft	Height from ground level to the top of the tree
H40	ft	The average total tree height for the 40 largest diameter trees/ac
HCB	ft	Height from ground level to the base of the compacted live crown

$H\text{-}D$	ft	Relationship of total tree height to diameter at breast height
I_{BCMF}	none	Indicator that data were from British Columbia Ministry of Forestry lands
I_{CR}	none	Indicator of a measured live-crown ratio
I_{FC}	none	Indicator of data measured on Forestry Canada plots
$k_{MT,j}$	none	Untreated tree calibration for all of the untreated plots on the j^{th} installation that included multiple thinning data with measurements of *CR*
KR	none	Correction to the mortality equation to place the number of trees and *QMD* on the maximum size-density line
$k_{SF,j}$	none	Untreated tree calibration for all of the untreated plots on the j^{th} installation that included single fertilization data
$k_{ST\&SF,j}$	none	Untreated tree calibration for all of the untreated plots on the j^{th} installation that included single thinning and single fertilized data with measurements of *CR*
$k_{ST,j}$	none	Untreated tree calibration for all of the untreated plots on the j^{th} installation that included single thinning data with measurements of *CR*
LOGS	none	Levels-of-Growing-Stock cooperative
LQ_i	ln(in.)	Natural logarithm of the *QMD* at the i^{th} measurement for a given number of trees
LT_i	ln(no./ac)	Natural logarithm of the number of trees at the i^{th} measurement
MLQ_i	ln(in.)	Natural logarithm of the maximum *QMD* at the i^{th} measurement for a given number of trees
$MTR_{\Delta D}$	none	Thinning-response modifier for 5-yr diameter increment combined across all multiply thinned plots on all installations
$MTR_{\Delta D,i,j}$	none	Thinning-response modifier for 5-yr diameter increment of the i^{th} tree growing on multiply thinned plots from the j^{th} installation
$MTR_{\Delta H}$	none	Thinning-response modifier for 5-yr height increment combined across all multiply thinned plots on all installations
$MTR_{\Delta H,i,j}$	none	Thinning-response modifier for 5-yr height increment of the i^{th} tree growing on multiply thinned plots from the j^{th} installation
NBT_i	no./ac	The number of trees removed in the i^{th} thinning
nf	count	The number of fertilizations on the plot
N_i	count	Number of trees/ac on the i^{th} plot
n_j	count	The number of trees with measured crown ratios on the j^{th} installation
NOb	none	Number of observations

NR_i	no./ac	The number of trees removed in the i^{th} thinning
nt	count	The number of thinnings on the plot
PCR_{SMC}	none	Live crown ratio predicted from the SMC *HCB* Eq. [1] or [2]
PCR_{VER}	none	Live crown ratio predicted from the "VER" version of the *HCB* equation (SMC or Z&H)
$PCR_{Z\&H}$	none	Live crown ratio predicted from the Zumrawi and Hann (1989) *HCB* equation
$P\Delta H$	ft	Potential 5-yr height increment of a tree
$P\Delta H40_C$	ft	Potential 5-yr change in *H40* on untreated plots
$P\Delta H_C$	ft	Potential 5-yr height increment of untreated trees
$P\Delta H_F$	ft	Potential 5-yr height increment of trees on fertilized trees
$PGEA$	yr	The age of the 40 largest-diameter trees with the same *H40* and the same *SI* as the subject plot
$PLEN$	5 yrs	Length of the growth period in 5-yr increments
PM	none	The probability of mortality during the next 5 yr
PN_i	lbs/ac	The weight of nitrogen applied/ac in the i^{th} fertilization
$Pred\Delta D_{C,i,j}$	in.	Predicted 5-yr diameter increment for the i^{th} tree growing on all untreated plots in the j^{th} installation that included the treatment of interest
$PredP\Delta H_{C,i,j}$	ft	Predicted potential 5-yr height increment for the i^{th} tree growing on all untreated plots in the j^{th} installation that included the treatment of interest
$Pred\Delta H$	ft	Predicted 5-yr change in *H*
$Pred\Delta H40_C$	ft	Predicted 5-yr change in *H40* on untreated plots
$PREM_{\Delta D}$	none	Proportion of *BABT* removed in past thinnings discounted by *YT* for the diameter growth data set
$PREM_{\Delta H}$	none	Proportion of *BABT* removed in past thinnings discounted by *YT* for the height growth data set
PS	none	Probability of survival (1.0 - *PM*)
QMD	in.	Quadratic mean diameter of the plot
QMD_B	in.	Quadratic mean diameter of the plot before the last thinning
QMD_T	in.	Quadratic mean diameter of the trees removed on the plot in the last thinning
RD_R	none	Reineke's (1933) relative density (SDI/SDI_{MAX})

RD_{RMOD}	none	RD_R thinning-response modifier
RH	ft	Reference height
SDI	Equivalent no. of 10 in. trees/ac	Reineke's (1933) stand-density index
SDI_{MAX}	Equivalent no. of 10 in. trees/ac	Maximum Reineke's (1933) stand-density index for a species
$SFR_{\Delta D}$	none	Fertilization-response modifier for 5-yr diameter increment combined across all singly fertilized plots on all installations
$SFR_{\Delta D,i,j}$	none	Fertilization-response modifier for 5-yr diameter increment of the i^{th} tree growing on singly fertilized plots from the j^{th} installations
$SFR_{\Delta H40}$	none	Fertilization response to the predicted 5-yr average dominant-height-growth equation combined across all singly fertilized plots on all installations
SI	ft	Site index
SI_{DF}	ft at 50 yr	Douglas-fir site index calculated from Bruce's 1981 dominant-height-growth equations
SI_{SP}	ft at 50 yr	Species-specific site index (SP = DF or WH)
SI_{WH}	ft at 50 yr	Western hemlock site index calculated from Bonner et al. (1995) site-index equations
SMC		Stand Management Cooperative
$ST\&SFR_{\Delta D}$	in.	Thinning- and fertilization-response modifier for 5-yr diameter increment combined across all singly thinned and fertilized plots on all installations
$STR_{\Delta D}$	in.	Thinning-response modifier for 5-yr diameter increment combined across all singly thinned plots on all installations
$STR_{\Delta D,i,j}$	in.	Thinning-response modifier for 5-yr diameter increment of the i^{th} tree growing on all singly thinned plots from the j^{th} installation
$STR_{\Delta H}$	ft	Thinning-response modifier for 5-yr height increment combined across all singly thinned plots from all installations
$TR_{\Delta D}$	none	Thinning-response modifier for 5-yr diameter increment combined across all thinned plots from all installations
$TR_{\Delta H40}$	none	Thinning response to the predicted 5-yr average dominant height growth equation combined across all thinned plots from all installations
VER	none	Version of height-to-crown-base equation used to predict crown ratio. VER = SMC for Eq. [1] or [2] and VER = $Z\&H$ for Zumrawi and Hann 1989

X_1	ln(in.)	Independent variable $\ln(D + 1.0)$
X_2	in.^2	Independent variable D^2
X_3	ln(ft)	Independent variable $\ln(SI_{SP} - 4.5)$
X_4	none	Independent variable $\ln[(CR + 0.2)/1.2]$
X_5	$\text{ft}^4/\ln(\text{in.})$	Independent variable $BAL^2/\ln(D + 5.0)$
X_6	ft	Independent variable $BA^{1/2}$
X_7	none	Independent variable $\ln[(PCR_{SMC} + 0.2)/1.2]$
X_8	none	Independent variable $(PREM_{AD}e^{\alpha_9 YT_i})$
X_9	ln(in.)	Independent variable $\ln(D + 5.0)$
X_{10}	in.	Independent variable D
X_{11}	$\text{ft}^2/\ln(\text{in.})$	Independent variable $BAL/\ln(D + 2.7)$
YF_i	yr	The number of years since the i^{th} fertilization was applied
YT_i	yr	The number of years since the i^{th} thinning was applied
Z_C	none	Portion of the probability of mortality due to untreated stand conditions
Z_F	none	Portion of the probability of mortality due to one or more fertilizations
Z_{FT}	none	Portion of the probability of mortality due to one or more fertilizations and one or more thinnings
Z_{SF}	none	Portion of the probability of mortality due to a single thinning
Z_T	none	Portion of the probability of mortality due to one or more thinnings