

Nonlinear Equations for Predicting Diameter and Squared Diameter Inside Bark at Breast Height for Douglas-fir

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Notice

This paper represents one chapter from the senior author's thesis, "Development of tree height and diameter growth equations for mid-Willamette Valley Douglas-fir," which will be submitted in partial fulfilment of the Master of Science Degree, Department of Forest Management, Oregon State University.

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Radial growth inside bark as measured from increment cores is often used in estimating past diameters. And change in squared diameter inside bark has been used in the development of equations for predicting basal area growth (Cole and Stage 1972). Because diameter outside bark is the variable most commonly measured, such measurements must often be converted to dimensions inside bark. In this paper regression equations are presented for predicting diameter inside bark at breast height and squared diameter inside bark for Douglas-fir on the basis of diameter outside bark. Application of these equations in studies of tree growth are discussed.

Background

Diameter Inside Bark

Many of the previous studies of the relationship between diameters inside and outside bark have been directed toward obtaining estimates of past diameter. Typically, a linear relationship between diameters inside and outside bark has been assumed, simplifying both the estimation procedure and application of results. Johnson (1955, 1956) and Spada (1960) used ordinary least squares regression to predict past diameter on the basis of double bark thickness:

$$BT = \beta_0^* + \beta_1^* \cdot DOB + \epsilon \quad (1)$$

where:

BT = double bark thickness

β_0^* , β_1^* = population parameter values

DOB = diameter outside bark

ϵ = a random error component with an expected value of zero and variance of σ^2 .

Model (1) can be rearranged to provide the model used by Finch (1948) and Dolph (1981) for estimating past diameters:

$$DIB = \beta_0 + \beta_1 \cdot DOB + \epsilon \quad (2)$$

where:

DIB = diameter inside bark

$\beta_0 = -\beta_0^*$

$\beta_1 = 1 - \beta_1^*$.

In working with model (2), Dolph (1981) also found that the residual variance exhibited heteroskedasticity. Consequently, he used weighted least squares regression to estimate the parameters.

Common to all of these studies was the inclusion of an intercept term. However, the presence of such a term can lead to questionable estimates of DIB for very small trees. These estimates are questionable because model (2) causes predicted DIB to exceed DOB for DOB's below $\beta_0/(1.0 - \beta_1)$. Because model (2) may be incorrectly specified for the full range of diameters, a more tenable form of the model might be

$$\text{DIB} = \beta_1 \cdot \text{DOB} + \epsilon. \quad (3)$$

Both Monserud (1979) and Powers (1969) have applied weighted least squares regression to fit this model, using weights of $1.0/\text{DOB}$ and $1.0/\text{DOB}^2$, respectively.

Finally, a nonlinear relationship between DIB and DOB for some species has been reported by Loetsch et al. (1973). They found the nonlinearity to be particularly pronounced for trees under 8 inches in diameter.

Squared Diameter Inside Bark

Squared diameter inside bark (or basal area inside bark) has been predicted by Cole and Stage (1972) and Monserud (1979) according to the following model:

$$\text{DIB}^2/\text{DOB}^2 = \beta_1 + \epsilon.$$

Because this model is the transformed version of a weighted least squares regression model, it can be restated as

$$\text{DIB}^2 = \beta_1 \cdot \text{DOB}^2 + \epsilon_1$$

where:

$$\text{Var}(\epsilon_1) = \sigma^2 \cdot \text{DOB}^4.$$

In the study described below, regression equations for DIB and DIB^2 will be used in developing estimators for past diameters and basal area growth inside bark as well as an equation for converting basal area growth inside bark to basal area growth outside bark.

The Study

Source of Data

This study was conducted as part of a larger project to develop a stand growth model for Oregon State University's McDonald-Dunn Forest. One element of this project is the development of equations for predicting tree diameter growth. These equations first require estimations of DIB and DIB^2 . Therefore, 724 Douglas-firs ranging in DOB from 4.1 to 43.0 inches were felled, and their diameters, diameter growth, and height growth were measured. The trees were selected to cover a range of stand conditions and site classes. On each tree, breast-height diameters inside and outside bark were measured to the nearest 0.1 inch according to both the long and the short axis of the cross section. At various times up to a year prior to felling, DOB of each tree had been measured with a diameter tape by a separate inventory crew.

Average diameters inside and outside bark were then calculated as the geometric means of their respective two measurements (Brickell 1976). This method provides unbiased estimates of basal area when the cross section is elliptical. The use of DOB calculated in this fashion as an independent variable in a predictive model does raise a problem. If errors-in-variables problems are to be avoided, the method used to measure the independent variable should be the same as that which would be used in the application of the model (Monserud 1976). Thus, the diameter-tape measurement would have provided a more suitable independent variable for regression analysis, but because of the intervening diameter growth, the geometric mean diameter was substituted. As a result, the models developed will be slightly biased when applied to trees measured with a diameter tape.

Estimating DIB

Model (3) was fitted to the data by weighted least squares regression; weights of $1.0/DOB$, $1.0/DOB^2$, and 1.0 (unweighted) were used. When the three weighting procedures were compared according to Furnival's (1961) index of fit, the index indicated that the weight of $1.0/DOB^2$ provided the best fit for the data. Residual plots from all three linear regressions revealed an unacceptable trend in the residuals as a result of forcing the regressions through the origin. If an intercept term is allowed in the equation, the trend in the residuals is eliminated and the intercept is significant ($P < 0.0001$). A plot of the data shows a strong linear relationship over the range of diameters sampled (Fig. 1). Unfortunately,

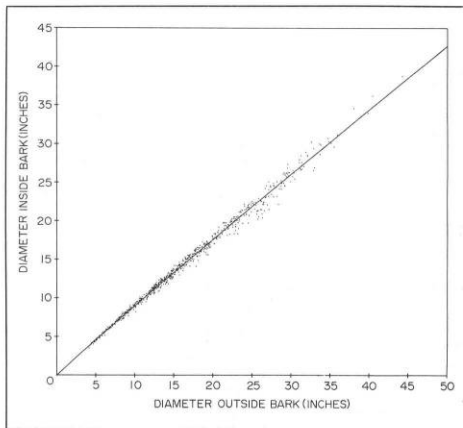


FIGURE 1.

REGRESSION OF DIAMETERS INSIDE BARK AT BREAST HEIGHT ON DIAMETERS OUTSIDE BARK FOR 724 DOUGLAS-FIRS ON McDONALD-DUNN FOREST. THE NONLINEAR MODEL (5) IS PLOTTED AS A SOLID LINE.

inclusion of the intercept term in model (2) results in unreasonable estimates for trees less than 2.0 inches in diameter.

In an effort to minimize this undesirable trend, two new models were tried. The first model

$$\ln(\text{DIB}) = \ln(\beta_1) + \beta_2 \cdot \ln(\text{DOB}) + \ln(\epsilon) \quad (4)$$

was fitted to the data by ordinary least squares regression. The second model

$$\text{DIB} = \beta_1 \cdot \text{DOB}^{\beta_2} + \epsilon_1 \quad (5)$$

was fitted by nonlinear regression with a weight of $1.0/\text{DOB}^2$. Algebraic manipulation of model (5) showed that DIB will exceed DOB for trees under 0.5 inch DOB, a distinct improvement over the performance of model (2). Also, the asymptotic estimate of the 99 percent confidence interval about β_2 for model (5) does not include 1.0; thus, the nonlinear trend in the model may be significant. Estimates of β_1 , β_2 , and Furnival's (1961) index of fit for models (3), (4), and (5) are found in Table 1. A plot of model (5) is found in Figure 1.

TABLE 1.

PARAMETER ESTIMATES AND FURNIVAL'S INDEX OF FIT FOR THE LINEAR, LOG-LINEAR, AND NONLINEAR MODELS OF DIAMETER INSIDE BARK.

Model	b_1	b_2	Index of fit
(3) Linear	0.887513	1.0	0.4507
(4) Log-linear ¹	.972252	.965836	.4017
(5) Nonlinear	.971330	.966365	.3952

¹ The parameter estimates for model (4) do not include a correction for bias introduced through the log transformation (Flewelling and Pienaar 1981).

Estimating DIB²

Analysis of the DIB data suggested that a nonlinear model would give the best fit to the DIB² data. Therefore, parameters were estimated by nonlinear regression with a weight of $1.0/\text{DOB}^4$ for the following model:

$$\text{DIB}^2 = \alpha_1 (\text{DOB}^2)^{\alpha_2} + \theta \quad (6)$$

where:

θ = a random component with expected value of zero and variance of $\sigma^2 (\text{DOB}^4)$.

Estimates of final parameters were 0.941944 for α_1 and 0.966843 for α_2 . Again, the asymptotic estimate of the 99 percent confidence interval about α_2 does not include 1.0.

Application of Results

DIB

While the applicability of these specific coefficients is limited by the range of the sample, the methodology employed in development and application is sufficiently general to be applied in other areas. The first application of the model for estimating DIB (model 5) is in the calculation of square-inch basal area increment inside bark (BAG_{IB}) on the basis of current diameter and radial growth inside bark:

$$\begin{aligned} BAG_{IB} &= (\pi/4) [DIB^2 - (DIB - 2 \cdot RG)^2] \\ &= \pi (RG) (DIB - RG) \end{aligned} \quad (7)$$

where:

RG = radial growth (inside bark, expressed in inches).

When model (5) is inserted into model (7), BAG_{IB} is then estimated as

$$BAG_{IB} = \pi (RG) [(0.971330 \cdot DOB^{0.966365}) - RG].$$

The second use of model (5) is in the estimation of past diameter. When the relationship between DIB and DOB is linear, the slope coefficient provides an indirect estimate of bark growth, as seen in the following expression for determining growth of DOB:

$$\begin{aligned} DIB &= b_0 + b_1 \cdot DOB \\ \Delta DIB &= b_1 \cdot \Delta DOB \\ \Delta DOB &= (1/b_1) \Delta DIB. \end{aligned}$$

Then:

$$\Delta DOB = (1/b_1) (2 \cdot RG).$$

Past diameter can then be expressed as

$$\begin{aligned} \text{DOB}_0 &= \text{DOB}_1 - \Delta\text{DOB} \\ \text{DOB}_0 &= \text{DOB}_1 - (2 \cdot \text{RG})/b_1 \end{aligned}$$

where:

DOB_0 = diameter at beginning of the growth period
 DOB_1 = current diameter.

The nonlinear relationship complicates the estimation procedure somewhat, but the estimate for DOB_0 can be derived, in a similar fashion, as follows:

$$\begin{aligned} \text{DIB} &= b_1 \cdot \text{DOB}^{b_2} \\ \Delta\text{DIB} &= b_1 (\text{DOB}_1^{b_2} - \text{DOB}_0^{b_2}). \end{aligned}$$

Therefore,

$$\text{DOB}_0 = (\text{DOB}_1^{b_2} - (2 \cdot \text{RG})/b_1)^{1/b_2}.$$

Then, when estimates of b_1 and b_2 from model (5) are used:

$$\text{DOB}_0 = [\text{DOB}_1^{0.966365} - (2 \cdot \text{RG}/0.971330)]^{1.03481}.$$

DIB²

The relationship described by model (6) is the basis for deriving predicted basal area outside bark (BA_2) as a function of current diameter and predicted basal area growth inside bark (BAG_{IB}):

$$\text{DIB}^2 = a_1 (\text{DOB}^2)^{a_2}.$$

Then:

$$\begin{aligned} \text{BAG}_{\text{IB}} &= (\pi/4) \Delta\text{DIB}^2 \\ &= (\pi/4) a_1 [(\text{DOB}_2^2)^{a_2} - (\text{DOB}_1^2)^{a_2}]. \end{aligned}$$

Therefore,

$$DOB_2^2 = [(4 \cdot BAG_{IB})/(\pi \cdot a_1) + (DOB_1^2)^{a_2}]^{1/a_2}$$

Inserting predicted basal area growth inside bark and converting the dependent variable to basal area gives the following:

$$\begin{aligned} BA_2 &= (\pi/4) [(4 \cdot BAG_{IB})/(\pi \cdot a_1) + (DOB_1^2)^{a_2}]^{1/a_2} \\ &= (\pi/4) [(4 \cdot BAG_{IB})/\pi(0.941944) + (DOB_1^2)^{0.966843}]^{1.03429} \end{aligned}$$

The Inverse Problem

The applications previously described imply inverse relationships of the form

$$DOB = (DIB/b_1)^{1/b_2}$$

and

$$DOB^2 = (DIB^2/a_1)^{1/a_2}$$

An alternative approach would be to derive separate regressions for DOB and DOB^2 in an approach analogous to that used by Myers and Alexander (1972). These two additional regressions were fitted to the data and found to provide estimates very close to those obtained by inverting models (5) and (6).

The closeness of these estimates was further checked by obtaining parameter estimates from models (5) and (6) and using them to compute the residual means and variances of the inverted forms. In both cases, the 95 percent confidence intervals about the means of the residuals included zero. Hence, there seems to be little practical difference between inverting the fitted equations or deriving two additional regressions. The method used in this paper does guarantee that estimated past diameters will be consistent with the current diameter measurement. That is, for trees with little or no radial growth, past diameter is constrained to be less than or equal to current diameter.

Conclusion

These data indicate that the ordinary least squares regression so frequently used in studies of this type may not always be applicable if one desires to characterize the full range of possible diameters. If estimates for small trees are desirable, then trees ranging from 0 to 8 inches in diameter should be strongly represented in the sample. This is the range of data most likely to exhibit nonlinearity.

Although a nonlinear model is best fitted to the data of this study, such model forms must be fitted to data from samples collected over a wider geographical area before conclusions can be drawn about their general applicability to Douglas-fir. The weighted nonlinear model does, however, merit further consideration in studies of this type.

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