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Height-Diameter Equations for Sixteen Tree Species in the Central Western Willamette Valley of Oregon

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Contents

Abstract

Equations for predicting tree height as a function of diameter outside bark at breast height are presented for 16 tree species from the central western Willamette Valley of Oregon. Foresters can use these "height–diameter" equations to avoid the time-consuming task of measuring heights of all individual trees in an inventory, a stand exam, or a timber cruise. Equation coefficients were estimated with weighted nonlinear regression techniques. Because site index can influence height, alternative equations including transformations of site index as an independent variable also are presented for 6 of these 16 species.

Introduction

Individual tree height is useful in assessing tree volume (Walters *et al.* 1985, Walters and Hann 1986), tree position within a stand (Ritchie and Hann 1986), and stand productivity through site index (Hann and Scrivani 1987), but it is time consuming to measure accurately. Instead, foresters now can use "height–diameter" equations to predict, rather than individually measure, tree heights. Traditionally, these equations have included only diameter as an independent variable (Curtis 1967). However, other stand-level variables, such as basal area and site index, may improve predictive capability (Larsen and Hann 1987). Foresters also can use these equations to indirectly estimate height growth by applying them to a sequence of diameters either measured directly in a continuous inventory or predicted indirectly by a diameter growth equation. The latter approach can be valuable for modeling growth and yield of trees and stands.

The objective of this study was to develop equations for predicting tree height as a function of diameter outside bark at breast height (DOB) by itself or in conjunction with site index for the tree species found in the central western foothills of the Willamette Valley:

Douglas-fir	*Pseudotsuga menziesii* (Mirb.) Franco
Grand fir	*Abies grandis* (D. Don) Lindl.
Pacific yew	*Taxus brevifolia* Nutt.
Ponderosa pine	*Pinus ponderosa* Laws.
Western hemlock	*Tsuga heterophylla* (Raf.) Sarg.
Western redcedar	*Thuja plicata* D. Don
Bigleaf maple	*Acer macrophyllum* Pursh

Oregon white oak	*Quercus garryana* Dougl.
Pacific dogwood	*Cornus nuttallii* Audubon
Bitter cherry	*Prunus emarginata* Eaton
Pacific madrone	*Arbutus menziesii* Pursh
Red alder	*Alnus rubra* Bong.
Oregon ash	*Fraxinus latifolia* Benth.
Willow	*Salix* spp.
Black hawthorn	*Crataegus douglasii* Lindl.
Oregon crab apple	*Malus fusca* (Raf.) Schneid.
Other hardwoods	

Data Description

The data for this analysis were part of the data base collected for developing a stand growth model for Oregon State University's McDonald and Dunn Research Forests located in the central western foothills of the Willamette Valley. All 136 stands selected had not been thinned or fertilized during the previous 5 years. Within each stand, a grid of 20 basal area factor (BAF) variable-radius plots was established so that the attributes of interest of all trees > 8 inches in DOB could be measured. Within each variable-radius plot, two fixed-area subplots were established so that the attributes of all trees ≤ 8 inches in DOB could be measured.

Trees with damaged tops or those shorter than breast height were excluded from the analysis. Tree DOBs were measured to the nearest 0.1 inch, tree heights were calculated to 0.1 foot, and site index of each stand was computed with King's (1966) site-index equations (Table 1). Because sample sizes for western redcedar, Oregon crab apple, and western hemlock are small, the use of predictor equations for these species must be approached with caution.

Data Analysis

Many of the height-diameter equations presented in the literature use a log-linear model form (Curtis 1967, Wykoff *et al.* 1982). However, other researchers have found that the residuals of these log-linear equations are not normally distributed (Larsen and Hann 1987), which makes correcting for log bias difficult

TABLE 1.

DESCRIPTIVE STATISTICS FOR THE HEIGHT–DIAMETER MODELING DATA SET.*

Species	Number of trees	DOB (in.)		Ht (ft)		SI (ft)	
		Mean (SE)	Range	Mean (SE)	Range	Mean (SE)	Range
Douglas-fir	12777	19.6 (0.13)	0.1–96.6	95.8 (0.41)	4.6–281.4	115.5 (0.10)	80.0–147.0
Grand fir	2218	10.6 (0.18)	0.1–48.3	60.8 (0.86)	4.6–216.9	115.9 (0.24)	83.0–137.0
Pacific yew	153	11.0 (0.53)	0.1–31.0	32.7 (1.01)	5.1– 63.6	117.6 (0.88)	83.0–136.0
Ponderosa pine	61	12.3 (0.53)	0.9–19.9	56.9 (1.90)	8.9– 83.6	112.1 (1.44)	94.0–147.0
Western hemlock	48	9.1 (1.22)	0.1–33.0	57.4 (4.36)	4.6–121.1	126.1 (0.80)	107.0–129.0
Western redcedar	12	17.1 (5.69)	0.1–61.6	55.7 (12.48)	4.6–120.6	120.7 (1.55)	108.0–129.0
Bigleaf maple	4014	10.2 (0.12)	0.1–52.1	53.8 (0.38)	4.6–139.1	115.3 (0.19)	83.0–147.0
Oregon white oak	1528	12.3 (0.21)	0.1–47.8	51.2 (0.53)	4.8–115.0	111.6 (0.32)	80.0–147.0
Pacific dogwood	803	2.5 (0.08)	0.1–18.1	22.2 (0.37)	4.6– 56.4	116.6 (0.33)	84.0–136.0
Bitter cherry	374	4.5 (0.12)	0.1–10.5	45.6 (1.07)	5.0– 94.4	119.6 (0.60)	83.0–136.0
Pacific madrone	335	8.6 (0.36)	0.1–45.3	43.4 (1.23)	4.6–118.4	112.6 (0.66)	80.0–147.0
Red alder	329	10.8 (0.35)	0.1–30.4	60.1 (1.15)	5.0–124.9	113.7 (0.65)	83.0–136.0
Oregon ash	245	7.9 (0.50)	0.1–42.8	45.6 (1.68)	4.9–121.0	99.3 (0.96)	83.0–137.0
Willow	107	5.2 (0.33)	0.1–17.1	31.6 (1.33)	4.6– 80.4	113.9 (1.00)	90.0–134.0
Black hawthorn	72	2.4 (0.28)	0.1– 9.1	19.5 (1.46)	4.6– 47.8	106.6 (1.71)	80.0–133.0
Oregon crab apple	34	3.8 (0.76)	0.2–20.8	18.0 (1.25)	5.5– 35.6	101.0 (2.02)	86.0–134.0
Other hardwoods	122	3.4 (0.39)	0.1–32.9	25.1 (1.68)	4.6–102.0	114.1 (1.09)	86.0–135.0

* DOB, diameter outside bark at breast height; Ht, total tree height; SI, King's (1966) site index; SE, standard error.

(Flewelling and Pienaar 1981). Therefore, the following nonlinear equation form was chosen for this study:

$$Ht = 4.5 + \exp(b_0 + b_1 DOB^{b_2}) \qquad [1]$$

where:

Ht = total tree height, feet

b_i = parameters to be estimated, $i = 0, 1, 2$

Larsen and Hann (1987) determined that the following more general equation form, which includes transformations of site index, explained more of the variation for some species:

$$Ht = 4.5 + \exp(b_0 + b_1 DOB^{b_2} + b_3 X) \qquad [2]$$

where X = additional independent variables that are transformations of site index. They also determined that the variances of residuals for Equations [1] and [2] were not homogeneous. Therefore, weighted regression was used to homogenize the variance. Comparing five different weights with Furnival's (1961) index of fit, Larsen and Hann (1987) found that a weight of 1.0/DOB was best.

For this study, Equations [1] and [2] were fit to each species-specific data set with weighted nonlinear regression techniques (weight = 1.0/DOB). Regression coefficients for both equations were checked with a t-test to see if they were significantly different ($p = 0.05$) from zero.

Results and Discussion

Tables 2 and 3 contain regression coefficients, weighted mean square errors, and weighted adjusted coefficients of determination for Equations [1] and [2] respectively.

For Equation [1], the coefficients were significantly different from zero for all species except ponderosa pine and Pacific yew. For both of these species, the slope coefficient of DOB, b_1, and the power coefficient of DOB, b_2, were so highly correlated that significant estimators of them could not be computed. Therefore, the estimate of b_2 was rounded to the nearest tenth and then

TABLE 2.

REGRESSION COEFFICIENTS AND ASSOCIATED WEIGHTED STATISTICS FOR THE MODELING DATA SET, EQUATION [1].

Species	Regression coefficients			Mean square error	Adjusted coefficient of determination
	b_0	b_1	b_2		
Douglas-fir	7.04524	−5.16836	−0.253869	13.9608	0.5308
Grand fir	7.42808	−5.80832	−0.240317	17.4615	0.6995
Pacific yew	9.30172	−7.50951	−0.100000	3.6000	0.2078
Ponderosa pine	6.10183	−4.48536	−0.300000	5.6573	0.3052
Western hemlock	4.80198	−2.78628	−0.692389	14.9939	0.7096
Western redcedar	6.18313	−4.41172	−0.283394	2.9967	0.9231
Bigleaf maple	5.21462	−2.70252	−0.354756	21.0577	0.2474
Oregon white oak	4.69891	−3.39164	−0.615259	11.5461	0.3383
Pacific dogwood	4.49727	−2.07667	−0.388650	12.6538	0.3548
Bitter cherry	7.91843	−5.45075	−0.181704	22.9442	0.5043
Pacific madrone	5.84487	−3.84795	−0.289213	14.8787	0.4054
Red alder	4.77965	−2.51761	−0.559775	12.2633	0.3560
Oregon ash	5.29159	−2.87443	−0.364288	14.0244	0.5896
Willow	4.88361	−2.47605	−0.309050	15.7936	0.1920
Black hawthorn	4.23548	−1.85815	−0.459519	9.6226	0.6410
Oregon crab apple	3.07101	−0.84120	−0.989203	8.9621	0.2689
Other hardwoods	5.98032	−3.70131	−0.249030	14.8670	0.5958

TABLE 3.

REGRESSION COEFFICIENTS AND ASSOCIATED WEIGHTED STATISTICS FOR THE MODELING DATA SET, EQUATION [2].*

Species	Regression coefficients				X_1	Mean square error	Adjusted coefficient of determination
	b_0	b_1	b_2	b_3			
Douglas-fir	6.19343	−5.34899	−0.239085	0.221197	ln(SI)	13.7456	0.5380
Grand fir	7.09574	−5.83136	−0.237430	0.315883E−2	SI	17.0813	0.7061
Ponderosa pine	5.66202	−4.51380	−0.300000	0.403337E−2	SI	5.2433	0.3561
Bigleaf maple	4.96819	−2.72538	−0.349626	0.234155E−2	SI	20.8609	0.2545
Bitter cherry	5.55308	−4.24198	−0.235550	0.991261E−2	SI	19.2375	0.5844
Willow	2.77188	−1.90051	−0.401561	0.139941E−1	SI	13.0206	0.3339

* X_1, additional independent variable; SI, King's (1966) site index; ln(SI), natural logarithm of site index.

fixed for each species, and b_0 and b_1 were refit; the resulting estimates were significantly different from zero.

Equation [2], which accounts for site index, provided a better fit for 6 of the 16 species. Of these 6, the only species with a nonsignificant coefficient (b_2) was ponderosa pine. That coefficient was fixed to the same value (–0.300000) as for Equation [1], and the remaining parameters were re–estimated.

In general, we recommend use of the equation with the greatest number of independent variables for the species. However, a potential user should consider, first, the expense or difficulty associated with collecting the site–index information needed to use Equation [2] and, then, whether the amount of additional variation explained by the more complicated equation is worth the cost or effort.

In sum, the equations developed in this study show that tree height is strongly correlated to diameter for the 16 tree species in this locale. In some cases, including site index in the equations improved the precision of predicting tree height. These height–diameter equations provide new and useful information about tree species growing in the central western foothills of the Willamette Valley of Oregon.

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British/Metric Conversion

1 inch = 2.54 centimeters
1 foot = 0.3048 meters

WANG, C.-H., and D.W. HANN. 1988. HEIGHT-DIAMETER EQUATIONS FOR SIXTEEN TREE SPECIES IN THE CENTRAL WESTERN WILLAMETTE VALLEY OF OREGON. Forest Research Laboratory, Oregon State University, Corvallis. Research Paper 51. 7 p.

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