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Equations for Predicting the 5-Year Height Growth of Six Conifer Species in Southwest Oregon

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Martin W. Ritchie

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Abstract

Equations for predicting individual-tree height growth per 5-year period are presented for Douglas-fir, white fir, grand fir, ponderosa pine, sugar pine, and incense-cedar growing in the mixed-conifer zone of southwest Oregon. The data used to develop the equations came from 3,648 trees sampled from 391 stands in the study area. Parameters were estimated by means of nonlinear regression. The model for height growth is expressed as a function of site index, total tree height, crown ratio, and percentage of crown closure at total height of the subject tree.

Introduction

Accurate and precise prediction of the growth and yield of forest stands is crucial for making informed decisions concerning the silvicultural treatment of stands and the allowable cut in forests, and for making informed estimates of the wood supply (Hann and Bare 1979, Hann and Brodie 1980). Increasingly, growth and yield information is coming from either whole-stand models such as DFSIM (Curtis *et al.* 1981) or single-tree models such as PROGNOSIS (Wykoff 1986, Wykoff *et al.* 1982) or CACTOS (Wensel *et al.* 1986). Most of the latter characterize a stand from a representative sample of individual trees for which diameter at breast height, total height, crown ratio, and an expansion factor (the number of trees per acre that a tree represents) have been measured or computed. A single-tree growth and yield model then uses diameter growth rate, height growth rate, change in crown ratio, and mortality equations to predict the composition and structure of the future stand.

In this paper, we give equations for predicting the 5-year height growth of the following conifer species:

Douglas-fir	*Pseudotsuga menziesii* (Mirb.) Franco
Grand fir	*Abies grandis* (Dougl. ex D. Don) Lindl.
White fir	*Abies concolor* (Gord. & Glend.) Lindl. ex Hildebr.
Ponderosa pine	*Pinus ponderosa* Dougl. ex Laws.
Sugar pine	*Pinus lambertiana* Dougl.
Incense-cedar	*Calocedrus decurrens* Torr.

The equations are used in the Southwest-Oregon version of ORGANON (ORegon Growth ANalysis and projectiON), a single-tree growth and yield model being developed for the mixed-conifer zone in that region (Hester *et al.* 1989).

The sample area, thus the area appropriate for application (Figure 1), extends from near the California border (42°10'N) on the south to Cow Creek (43°00'N) on the north, and from the crest of the Cascade Mountains (122°15'W) on the east to approximately 15 miles west of Glendale, Oregon (123°50'W). Elevation ranges from 900 to 5,100 feet. January mean minimum temperatures range from 23° to 32°F and July mean maximum temperatures from 79° to 90°F. Annual precipitation varies from 29 to 83 inches, with less than 10 percent falling during June, July, and August.

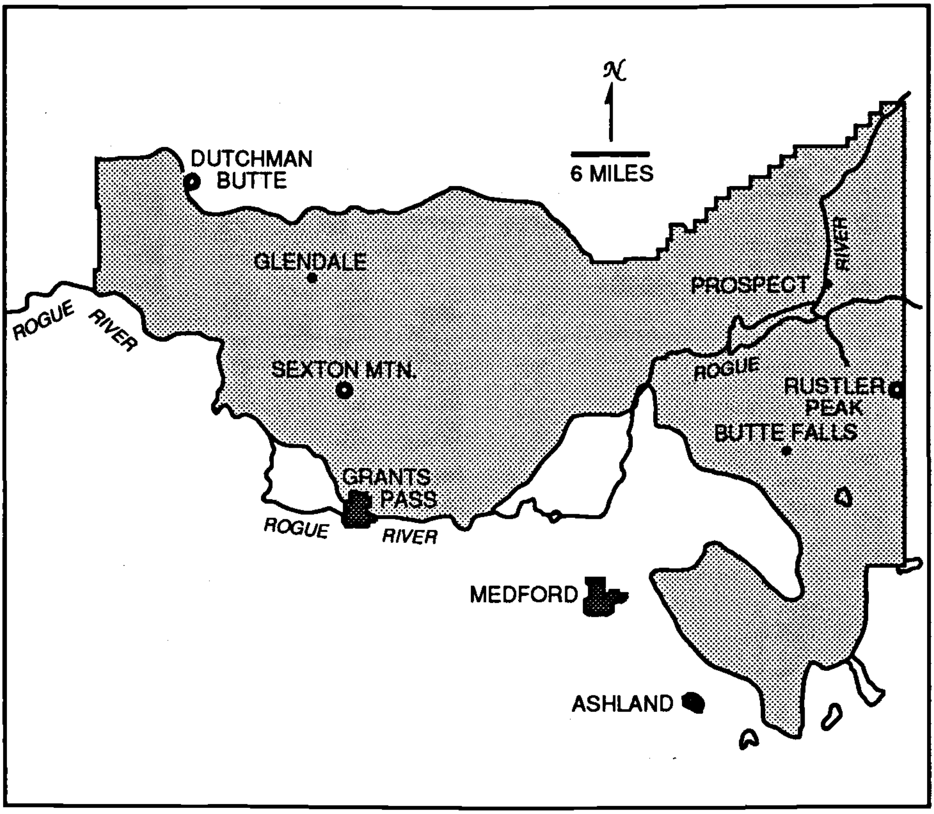

Figure 1. Map of the southwest-Oregon study area.

Data

The data were collected from temporary plots established in 391 stands during 1981, 1982, and 1983 as part of the southwest Oregon Forestry Intensified Research (FIR) Growth and Yield Project. In each stand, a cluster of from four to ten variable-radius plots and two nested fixed-area subplots were established for measurement of the attributes of all trees ≥6 inches tall. A circular fixed-area subplot with a 7.78-foot radius (1/229th acre) was used for trees with outside-bark diameter at breast height (DBH) ≤4 inches, a circular fixed-area subplot with a 15.56-foot radius (1/57th acre) for trees with 4.1- to 8-inch DBH, and a variable-radius plot (basal-area factor 20) for trees with DBH ≥8.1 inches.

Data taken for individual trees at the end of the 5-year growth period were: mortality within the period, DBH_2, total height (H_2), height to live-crown base (HCB_2), and horizontal distance from plot center to tree center (DIST). (The subscript "2" in these representations indicates end of the growth period.) In addition, past 5-year radial growth and height growth were measured on subsamples of the trees.

The dating of mortality was based upon physical features of the dead tree as described by the USDA Forest Service (1978) and Cline *et al.* (1980). Breast-height diameter was measured to the nearest 0.1 inch with a diameter tape. Both height measurements were taken by the tangent method (Curtis and Bruce 1968, Larsen *et al.* 1987). The position of the crown base was determined by visually reconstructing the crown such that any gaps in it were mentally filled with branches from below to produce a symmetrical, even base. The horizontal distance from plot center to tree center was determined by adding the distance from the center to the tree face and one-half DBH_2, expressed in feet.

Past 5-year radial growth rate at breast height was measured on all trees large enough to accept an increment borer. Past 5-year height growth of all undamaged trees <25 feet tall with distinguishable internode lengths was measured with a 25-foot telescoping pole. A subsample of as many as six trees ≥25 feet tall that were selected to cover the range of DBH and crown class in each stand was felled on each plot. The ages of these at the first and sixth whorls were determined to ensure a true 5-year growth period, and the distance between the two whorls was measured for 5-year height growth.

An expansion factor, or number of trees per acre represented by a live tree at the end of the growth period ($EXPAN_2$), was then computed from the tree DBH_2 and the rules appropriate for the sampling design.

Because the project objective was to estimate future rather than past height growth, all tree measurements were backdated to values for the start of the previous 5-year growth period, i.e. DBH_1, H_1, HCB_1, and $EXPAN_1$ were estimated. The procedures for backdating measurements for living trees are given in Hann and Wang (1990). It was assumed that trees that died during the growth period had the same values at the start and end of the period.

Once the basic variables were backdated, variables for the sample tree and stand were calculated. Crown ratio (CR) at the start of the growth period was determined by:

$$CR_1 = 1.0 - \frac{HCB_1}{H_1}.$$

The percentage of crown closure at total tree height at the start of the growth period (CCH_1)was calculated to quantify tree position within a stand. In calculating CCH_1 of a particular tree, the total height of the tree at the start of the growth period (H_1)was used as a reference height. First, crown width of each tree at the reference height was estimated with the crown-width relationships of Ritchie and Hann (1985) and the equations for maximum crown width of Paine and Hann (1982). If the reference height fell above the tree tip, crown width was zero; if it fell below crown base, crown width at crown base was used. Next, each value was converted to crown area by the formula for the area of a circle. The values were then summed and expressed as a percentage. Finally, the stand site index (S) was calculated with the equations of Hann and Scrivani (1987). The data are summarized in Table 1.

Table 1. Data summary for development of the height growth modifier for six conifer species: Douglas-fir (DF), grand fir and white fir (TF), ponderosa pine (PP), sugar pine (SP), and incense-cedar (IC).

		Five-year height growth		Crown ratio		Crown closure at total tree height		Total height	
Species	Sample size	Range	Mean	Range	Mean	Range	Mean	Range	Mean
DF	2483	0.3-18.7	6.44	0.01-1.00	0.67	0-174.1	27.5	4.6-203.2	45.1
TF	546	0.3-22.5	5.50	0.07-1.00	0.66	0-172.8	31.6	4.6-167.4	41.3
PP	169	1.1-14.3	6.01	0.15-1.00	0.48	0-60.8	8.9	25.0-171.1	79.9
SP	118	0.5-11.0	6.09	0.21-1.00	0.58	0-88.2	11.0	5.5-168.6	73.6
IC	332	0.1-10.8	3.28	0.10-1.00	0.65	0-147.5	44.3	4.6-123.4	25.9

Analysis

The model form used in this study has been previously reported by Hann and Ritchie (1988). In brief, it starts by establishing the maximum potential height growth for each tree for a 5-year period. This is predicted with the assumption that the tree is a dominant one. The value is then multiplied by a modifier function that reflects the impact of stand structure and individual tree vigor on height growth.

Potential 5-Year Height Growth

Potential 5-year height growth is derived from equations for height growth of dominant trees (Hann and Scrivani 1987). Because some stands in this study were of uneven age, the growth effective age (GEA), a function of site index and current total height (H) of the given tree, was used in place of the stand-age variable in the equation

$$\mathrm{PHG} = f(\mathrm{S}, \mathrm{GEA}) - \mathrm{H},$$

where

PHG = potential 5-year height growth (in feet), and

f(S,GEA) = dominant height for S and GEA

$$= 4.5 + [\mathrm{H} - 4.5]\left\{\frac{1 - \exp\left[a_0(\mathrm{S} - 4.5)^{a_1}(\mathrm{GEA} + 5)^{a_2}\right]}{1 - \exp\left[a_0(\mathrm{S} - 4.5)^{a_1}(\mathrm{GEA})^{a_2}\right]}\right\}$$

in which

$$\mathrm{GEA} = \left[\frac{\ln\left(1 - \left\{\frac{\mathrm{H} - 4.5}{\mathrm{S} - 4.5}\right\}\left\{1 - \exp\left[a_0\,(\mathrm{S} - 4.5)^{a_1}\,50^{a_2}\right]\right\}\right)}{a_0(\mathrm{S} - 4.5)^{a_1}}\right]^{1/a_2}, \text{ and}$$

a_0, a_1, a_2 = regression coefficients.

Figure 2 illustrates the method of computing the potential 5-year height growth of a dominant Douglas-fir tree growing on a site with a given index. Growth effective age is determined by extending a horizontal line from the y-axis value for tree height at the start of the given growth period to the curve for the site index, then dropping a vertical line to the x-axis. For an estimate of dominant tree height at the end of the growth period, 5 years is added to the x-axis value for growth effective age, a vertical line is projected back to the curve for the site index, and a horizontal line is extended to the y-axis. The difference between this point and the starting height is the potential height growth.

This method assumes that shorter trees that hold dominant positions in the stand are younger and may grow faster than their taller counterparts; therefore, "dominance" of a given tree is defined in relation to its position with respect to its immediate neighbors. Dominant trees in a stand may have substantially different height, provided that the stand is uneven aged or contains an opening in the canopy through which a smaller tree may express dominance.

Hann and Scrivani (1987) found that the ponderosa pine site index was 0.941 of the Douglas-fir site index on the same site and that

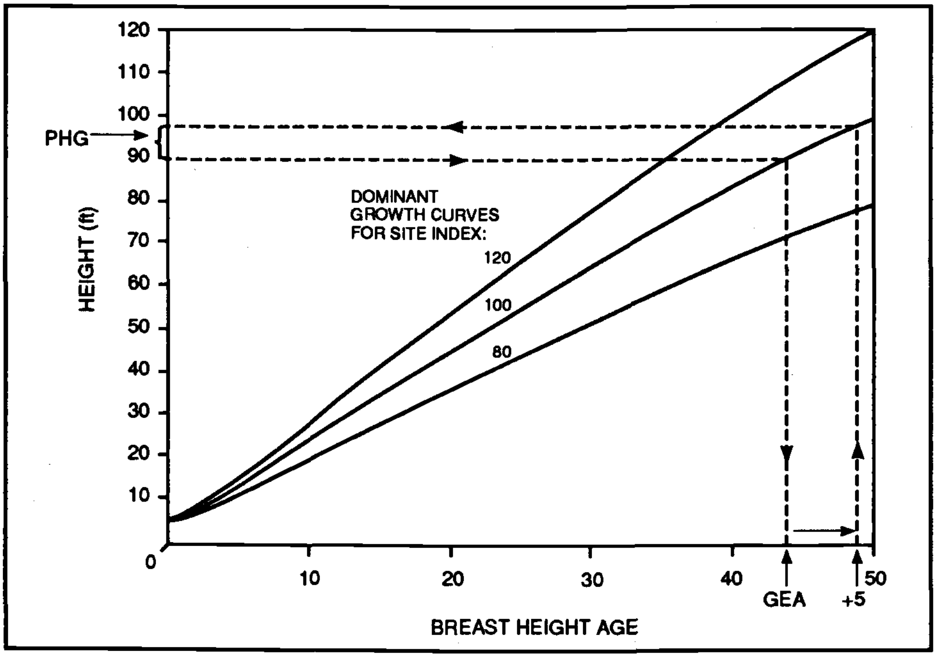

Figure 2. An example of plotting the growth effective age (GEA) and potential height growth (PHG) of dominant Douglas-fir trees growing on land with the site index 100.

the dominant height growth of ponderosa pine differed from that of Douglas-fir. For that reason, they developed separate equations for the dominant height growth and site index of ponderosa pine, but they lacked the data needed for species-specific equations for height grand fir, white fir, and sugar pine. We therefore used the equation for Douglas-fir dominant height growth to estimate the potential height growth of these species. On those plots that supported dominant trees of more than one species, the dominant height growth of Douglas-fir was strongly congruent with that of grand fir, white fir, and sugar pine. We found no evidence of species-dependent departures in height-growth or site-index estimates obtained with the Douglas-fir equation.

The Douglas-fir equation for dominant height growth was also used to determine the potential height growth of incense-cedar. However, the site index for that species was only about 70 percent of that for Douglas-fir in any given stand (i.e., a dominant incense-cedar on the same site as a dominant Douglas-fir would be approximately 70 percent as tall at age 50). Again, Hann and Scrivani (1987) lacked the data needed to develop the appropriate curves; therefore, we multiplied the average site index value for Douglas-fir by 0.70.

The expression for potential height growth in the function is multiplied by a scalar we call P_0. This parameter reflects the difference between the average growth of dominant trees and the maximum potential growth. For all species, we found P_0 to be approximately 1.06, which indicates that the tallest trees in the stand are growing faster than the function for potential height growth would predict. We attribute this to the fact that site index is computed from an average value for dominant trees measured in the stand. Because some trees will have site-index values that are higher than average, they will have greater potential height growth than would be predicted. If P_0 is not included in the model, then the predicted average height growth of dominant trees will be below the true average potential.

Modifier Function

Of the variables considered as a measure of stand density and tree position, crown closure at total tree height (CCH) was selected as best because it minimized the mean squared error for height growth. The value for CCH is an estimate of the percentage of crown closure affecting a tree at its top. A tree with a zero CCH has no taller competitors. In an even-aged stand, a tree with CCH near zero is usually dominant. A tree with CCH greater than 100 is growing beneath a closed canopy and would likely be rated as suppressed, or perhaps intermediate, as it receives no light from above. The CCH value will always be less than or equal to the percentage of crown closure of the entire stand.

The relationship of CCH to crown class is not as well defined in multi-storied or uneven-aged stands. In these stands, a tree with high CCH may be growing in a local opening. However, such a tree would probably have a longer crown than one with the same CCH that is growing in the understory of an even-aged stand. Crown ratio, therefore, is a useful index of local growing conditions and, therefore, of tree vigor.

A detailed statistical and graphical analysis by Hann and Ritchie (1988) of the Douglas-fir data used in this study revealed that model forms presented in other studies (Arney 1985, Ritchie and Hann 1986, Wensel *et al.* 1987) were not sufficiently flexible to reflect the trends shown when 5-year height growth is plotted over crown ratio by different classes of CCH. Therefore, they used the techniques described by Jensen (1984) to derive an alternative modifier function.

The height-growth equation developed by Hann and Ritchie (1988) is:

$$HG = P_0\,(PGH)\,(MOD) \qquad [1]$$

where

HG = 5-year height growth (in feet), and

MOD = the modifier function

$$= \left\{F_0 + (F_1 - F_0)\exp\left[-F_2\,(1 - CR)^{P_6}\right]\right\},$$

in which

$$F_0 = P_1 \exp(P_2\,CCH),$$

$$F_1 = \exp\left(P_3\,CCH^{P_4}\right),$$

$$F_2 = P_5 \exp\left(P_7\,CCH^{P_4}\right),$$

P_0, P_1, ..., P_7 = the model parameters,

CR = live crown length divided by total height, and

CCH = crown closure at total tree height (in percent).

Nonlinear least squares analysis with the iterative procedure proposed by Marquardt (1963) was used to obtain the parameter estimates for equation [1]. As in earlier modeling efforts in southwest Oregon, the true firs (grand fir and white fir) were assumed to have identical response surfaces (Larsen and Hann 1987, Ritchie and Hann 1987); therefore the data for these two species were combined.

The model is, to a certain degree, over-specified with respect to the data in this study, but because the graphical technique used in the first phase of the analysis provided good estimates for many of the parameters, it presents no problem. We fixed parameters P_4, P_5, and P_6 at the graphical estimate.

Results and Discussion

The parameter estimates for the potential-height-growth function (from Hann and Scrivani 1987) are:

	a_0	a_1	a_2
Douglas - fir	-0.00199536	0.281176	1.14354
Ponderosa pine	-0.00143431	0.288169	1.21297

The parameter estimates for the modifier functions (as well as P_0, a component of the potential function) are shown in Table 2 and in graphs of the modifier functions in Figure 3. The solid lines in the figure represent the functions within the range of the data. It is apparent that the modifier reaches a peak when the crown ratio is near 1.0. Reductions in height growth rate are not independent of density. For species other than ponderosa pine, trees in a dominant position (CCH near zero), have a predicted height growth rate that differs little across the range of crown ratios. However, as CCH increases, competitive stress increases, and shorter crowned trees may show a substantial reduction in height growth.

Ponderosa pine (Figure 3) shows a more pronounced effect of crown ratio at lower CCH values, which supports the findings of Lynch (1958) and Oliver (1967) that density significantly affects height growth of dominant trees. The modifier functions for the remaining species show a similar trend (Figure 3): a peak at 1.0 and a decrease in growth with increasing CCH and decreasing crown ratio.

Parameter P_1 was fixed at 1.0 for incense-cedar and true fir because it was not significantly different from 1.0 ($\alpha=0.01$). Further-

Table 2. Parameter estimates (with standard errors in parentheses) and the mean squared error (MSE) of the height growth modifier for six conifer species of southwest Oregon: Douglas-fir, grand fir and white fir (true fir), ponderosa pine, sugar pine, and incense-cedar .

Parameters	Douglas-fir	True fir	Ponderosa pine	Sugar pine	Incense-cedar
P_0	1.0644	1.0644	1.0650	1.0644	1.0644
P_1	0.876948 (0.0369)	1.0	0.752392 (0.114)	0.890906 (0.0860)	1.0
P_2	-0.0365001 (0.00361)	-0.0328142 (0.00243)	-0.0662973 (0.0272)	-0.0275987 (0.0113)	-0.0375305 (0.00495)
P_3	-0.0506209 (0.00187)	-0.0127851 (0.000904)	-0.00674461 (0.00221)	-0.00835840 (0.00237)	-0.0156989 (0.00103)
P_4[a]	0.5	1.0	1.0	1.0	1.0
P_5[a]	1.5625	6.1978	2.3704	2.7778	2.7778
P_6[a]	2.0	2.0	3.0	2.0	2.0
P_7	0.136986 (0.0125)	0.0	0.0	0.0	0.0
MSE	4.8440	5.6643	3.7352	4.0163	2.7054

[a] Parameters not estimated by regression are discussed in text.

more, P_7 could be fit only for Douglas-fir. For other species, either P_7 was not significant, or it caused the regression routine to cycle without reaching convergence. This parameter serves to change the shape of the response surface (moving the inflection point) with change in CCH. For Douglas-fir, the difference is subtle; for the other species, the data sets are not large enough to show the trend.

It should be noted that the modifier functions have not been conditioned to pass through zero when the crown ratio is zero. As a result, the height growth predicted for a tree with a crown ratio close to zero may be substantial if the CCH value is also small. A crown ratio of zero means that a tree is dead; a small CCH value and a very small crown ratio may mean that the tree has suffered disease or severe insect infestation, or the top may have been removed by wind or ice. These conditions are beyond the scope of this study, and the equations should not be applied. For example, only two trees had a crown ratio less than 0.08 in the Douglas-fir data set. Trees with a crown ratio less than 0.15 and CCH less than 40 should be suspect;

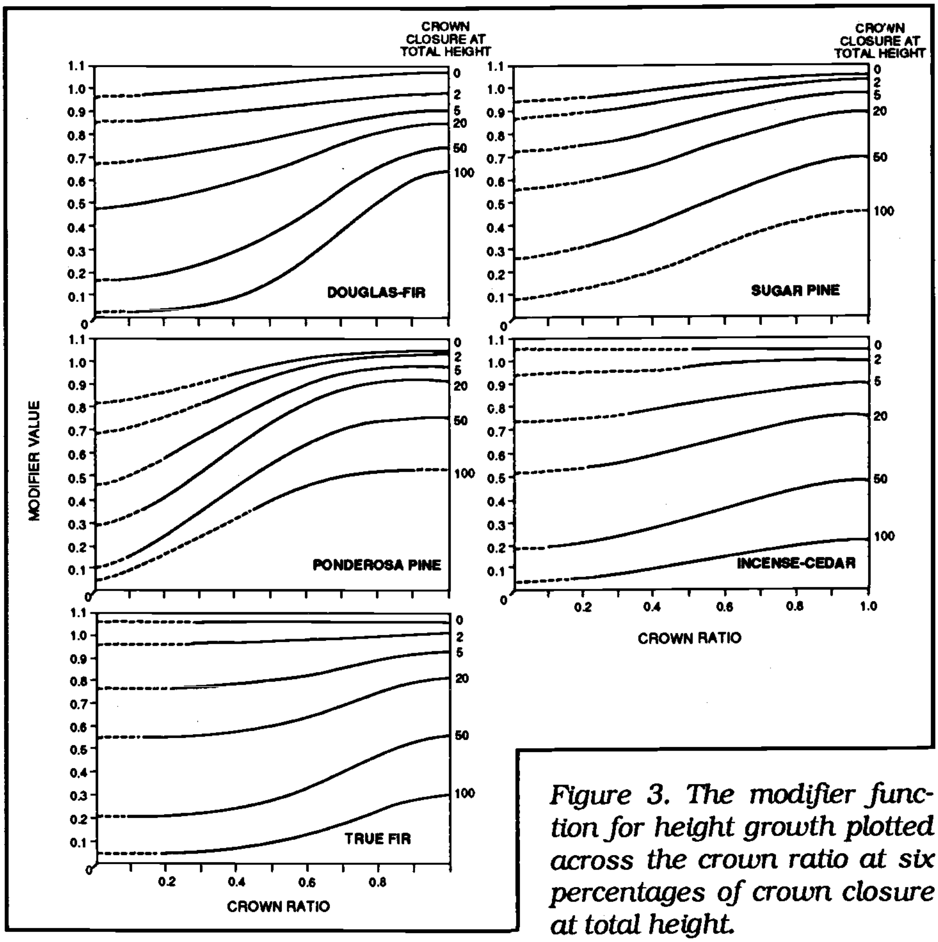

Figure 3. The modifier function for height growth plotted across the crown ratio at six percentages of crown closure at total height.

the model will not work well for them. The solid lines in Figure 3 represent the approximate range of the data at different levels of CCH. They may be used for determining appropriate ranges of CCH and crown ratio for these height-growth models.

Finally, it should be remembered that the data used to develop these equations came predominantly from untreated stands. Therefore, application of the equations to thinned stands or stands that have undergone overstory removal will be an extrapolation. Results with such applications should be viewed cautiously.

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